1. Let \( z = \cos \theta + i \sin \theta \), \( 0 < \theta < \frac{1}{2} \pi \). Find the modulus and argument of \( 1 - z \).

2. Let \( n \) be a positive integer. Show that \( \cos(n\theta) \) can be expressed as a polynomial in \( \cos \theta \) in which the coefficient of \( \cos^n \theta \) is \( 2^{n-1} \).

3. If \( m \) is a positive integer, prove that

\[
\begin{align*}
 z^{2m} - 1 &= (z - 1)(z + 1) \prod_{k=1}^{m-1} \left( z^2 - 2z \cos \frac{k\pi}{m} + 1 \right), \\
 z^{2m+1} - 1 &= (z - 1) \prod_{k=1}^{m} \left( z^2 - 2z \cos \frac{2k\pi}{2m+1} + 1 \right).
\end{align*}
\]  

Deduce from (0.1) that

\[ \prod_{k=1}^{m} \sin \frac{k\pi}{2m} = \frac{\sqrt{m}}{2^{m-1}}. \]

4. Find the principal value of \( i^i \).

5. Find all solutions to

\[ \exp(e^z) = 1. \]

6. Although \( z \) and \( \overline{z} \) are not independent variables, we shall formally treat them as if they were. Define

\[ f(z, \overline{z}) := u(x, y) + iv(x, y). \]

Show that

\[
\begin{align*}
\frac{\partial f}{\partial z} &= \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{i}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right), \\
\frac{\partial f}{\partial \overline{z}} &= \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) + \frac{i}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right).
\end{align*}
\]

Hence, show that the Cauchy-Riemann equations are equivalent to

\[ \frac{\partial f}{\partial \overline{z}} = 0. \]