Math 535
Homework I

Due Wed. Jan. 28

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Problem 1. Find all topologies on the set $X = \{0, 1, 2\}$.

Problem 2. (i) Show that if X is a set equipped with a trivial topology and Y is any topological space, then *every* function $f : Y \to X$ is continuous.

(ii) Show that if X is a set equipped with a discrete topology and Y is any topological space, then *every* function $f : X \to Y$ is continuous.

Problem 3. Use de Morgan's laws to prove

(i) The union of finitely many closed subsets of a topological space is closed, and

(ii) The intersection of arbitrarily many closed subsets of a topological space is closed.

Problem 4. Show that if X is a set and a collection σ of subsets of X is given, satisfying the conditions C1)-C3) given in class, then the collection $\tau = \{U \subseteq X \mid X \setminus U \in \sigma\}$ is a topology on X.

Problem 5. Given an example of a topological space and a collection $\{W_\alpha\}_{\alpha\in A}$ of closed subsets such that their union $\bigcup_{\alpha\in A} W_\alpha$ is *not* closed.

Problem 6. Let $\mathbb{Q} \subseteq \mathbb{R}$ be the subset of rational numbers. Show that $\mathbb{Q}$ is neither open nor closed.

Problem 7. (The cofinite topology) Let X be any set and define a nonempty subset $U \subseteq X$ to be open if $X \setminus U$ is finite. Show that this defines a topology on X.

Problem 8. Let $\mathscr{B}$ be a basis for a topology on X and define a subset $U \subseteq X$ to be open, as it was in class, if for every $x \in U$ there is $V \in \mathscr{B}$ such that $x \in V \subseteq U$. Show that this satisfies the definition of a topology.