

225 N1 EXAM 1

Name:

Directions: Answer all questions. Be neat. To earn full credit for each problem, you must show work and justify statements when asked. No calculators. Numbers in () are the values of each problem.

1. Find the general solution for the linear system whose augmented matrix is (10)

$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

2. Mark each statement true or false. Justify each answer. (12)
- If a system of linear equation has two different solutions, it must have infinitely many solutions.
 - If \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} in \mathbb{R}^n , then \mathbf{u} is a linear combination of \mathbf{v} and \mathbf{w} .
 - The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if there are no free variables.
 - If A and B are $n \times n$ matrices and $AB = 0$ then either $A = 0$ or $B = 0$.

3. For the matrix below, find its inverse. Is every \mathbf{b} in \mathbb{R}^3 in the span of the columns of A ? (14)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 0 & -3 & 1 \end{bmatrix}$$

4. For matrices E and F below, for each of EF and FE state whether it is defined, and if it is then compute it. (12)

$$E = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad F = \begin{bmatrix} -3 & 2 & 1 \\ -1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A as a_1 , a_2 and a_3 and set $W = \text{Span}(a_1, a_2, a_3)$. This problem has two parts.

i) is \mathbf{b} in W ? (10)

ii) Show that a_1 is in W . Use no row operations. (4)

6. Given the matrix $M = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}$, observe that the third column is the sum of the first two columns. Find a nontrivial solution of $M\mathbf{x} = \mathbf{0}$ without using any row operations. (12)

7. Describe the general solution of the following system in parametric vector form. (14)

$$\begin{aligned}x_1 + 3x_2 + x_3 &= 1 \\-4x_1 - 9x_2 + 2x_3 &= -1 \\-3x_2 - 6x_3 &= -3\end{aligned}$$

8. Suppose my economy is divided into three parts, manufacturing, agriculture, and services. For each unit of output, manufacturing requires .10 unit from other companies in that sector, .30 unit from agriculture, and .30 unit from services. For each unit of output, agriculture uses .20 unit of its own output, .60 unit from manufacturing and .10 unit from services. For each unit of output, the services sector consumes .10 unit from services, .60 unit from manufacturing, but no agricultural products. Determine the production levels needed to satisfy a final demand of 18 units for manufacturing, 20 units for agriculture, and 30 units for services. Just set up the calculation. Do no row operations for this problem. I want you to write down an augmented matrix that you would need to row reduce to answer this question. (12)