9am

MIDTERM 1 SOLUTIONS

Question 1

(a) $$x_1\begin{bmatrix}1\\2\end{bmatrix}+x_2\begin{bmatrix}2\\5\end{bmatrix}+x_3\begin{bmatrix}0\\1\end{bmatrix}+x_4\begin{bmatrix}-1\\0\end{bmatrix}=\begin{bmatrix}2\\6\end{bmatrix}$$

(b) $$\begin{bmatrix}1&2&0&-1\\2&5&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}2\\6\end{bmatrix}$$

(c) Row reduce the augmented matrix of coefficients:

$$\begin{bmatrix}1&2&0&-1&2\\2&5&1&0&6\end{bmatrix}\longrightarrow\begin{bmatrix}1&2&0&-1&2\\0&1&1&2&2\end{bmatrix}$$

$$\longrightarrow\begin{bmatrix}1&0&-2&-5&-2\\0&1&1&2&2\end{bmatrix}$$

$$\Longrightarrow\quad \begin{aligned}x_2&=2-x_3-2x_4\\ x_1&=-2+2x_3+5x_4\end{aligned}$$

So $$\boxed{\vec{x}=\begin{bmatrix}-2\\2\\0\\0\end{bmatrix}+x_3\begin{bmatrix}2\\-1\\1\\0\end{bmatrix}+x_4\begin{bmatrix}5\\-2\\0\\1\end{bmatrix}}$$

(d) both set $= \text{Span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

i.e. the solution set is the plane spanned by the vectors $\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -2 \\ 0 \\ 1 \end{bmatrix}$.

Question 2

(a) $\begin{bmatrix} 3 \\ -6 \\ 0 \\ 9 \\ 12 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \\ 4 \end{bmatrix}$. Hence the 2 vectors are linearly dependent

(b) Row reduce:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 0 & 1 \\ 0 & 12 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 6 & 3 \\ 0 & 12 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 6 & 3 \\ 0 & 0 & -5 \end{bmatrix}$$

Since the echelon form has a pivot in every column, there are no non-trivial solutions to $A\vec{x} = 0$. Thus there are no non-trivial linear combinations $x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 0 \\ 12 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$.

That is, the vectors are linearly independent.

Question 3

Use row reduction:

$$[A\ I] = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 6 & -2 & 1 & 2 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1/3 & 1/6 & 1/3 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 0 & 4/3 & -1/6 & -1/3 \\ 0 & 1 & 0 & 2/3 & 1/6 & -2/3 \\ 0 & 0 & 1 & -1/3 & 1/6 & 1/3 \end{bmatrix} \qquad \begin{pmatrix} r_1 \to r_1 - r_3 \\ r_2 \to r_2 - 2r_3 \end{pmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & +2/3 & -1/3 & 1/3 \\ 0 & 1 & 0 & 2/3 & 1/6 & -2/3 \\ 0 & 0 & 1 & -1/3 & 1/6 & 1/3 \end{bmatrix} = [I, A^{-1}]$$

$$\boxed{A^{-1} = \begin{bmatrix} 2/3 & -1/3 & 1/3 \\ 2/3 & 1/6 & -2/3 \\ -1/3 & 1/6 & 1/3 \end{bmatrix}}$$

Question 4

(a)	F
(b)	T
(c)	T
(d)	F
(e)	T
(f)	F
(g)	T
(h)	T
(i)	F
(j)	F