

MATH 225 - Assignment 1

[Solutions to even # problems, Section 1.1]

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The next two steps:

1. Clear out the non-zero terms below the pivot in column 3. Replacing row 3 by $(\text{row } 3) - 3(\text{row } 2)$ yields

$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -5 & 15 \end{bmatrix}$$

2. Either solve by back substitution or proceed to get reduced echelon form. For the latter, the next step would be to make the pivot in column 4 equal to 1 (by multiplying row 4 by $-\frac{1}{5}$).

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Get matrix into reduced echelon form:

$$\begin{bmatrix} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (r_3 \rightarrow \frac{1}{2}r_3)$$

$$\rightsquigarrow \begin{bmatrix} 1 & -4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{cases} r_2 \rightarrow r_2 - 7r_3 \\ r_1 \rightarrow r_1 - 9r_3 \end{cases}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (r_1 \rightarrow r_1 + 4r_2)$$

Thus the solution is

$$\begin{array}{l} x_3 = 0 \\ x_2 = 0 \\ x_1 = 0 \end{array}$$

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$$4T_1 - T_2 - T_4 = 30$$

$$4T_2 - T_1 - T_3 = 60$$

$$4T_3 - T_2 - T_4 = 70$$

$$4T_4 - T_1 - T_3 = 40$$

The augmented matrix is thus

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

$$\begin{array}{l} r_2 \leftrightarrow r_1 \\ (\text{new}) r_1 \rightarrow -r_1 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix}$$

$$\begin{array}{l} r_2 \rightarrow r_2 + r_1 \\ r_4 \rightarrow r_4 - 4r_1 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & 0 & 16 & -8 & 300 \\ 0 & 0 & -16 & 56 & 780 \end{bmatrix}$$

$$r_3 \rightarrow 4r_3 + r_2$$

$$r_4 \rightarrow 4r_4 + r_2.$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & 0 & 16 & -8 & 300 \\ 0 & 0 & 0 & 48 & 1080 \end{bmatrix}$$

$$r_4 \rightarrow r_4 + r_3.$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 1 & -1/2 & 75/4 \\ 0 & 0 & 0 & 1 & 45/2 \end{bmatrix}$$

$$r_2 \rightarrow \frac{1}{4} r_2$$

$$r_3 \rightarrow \frac{1}{16} r_3$$

$$r_4 \rightarrow \frac{1}{48} r_4$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 55/2 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 45/2 \end{bmatrix}$$

$$r_1 \rightarrow r_1 + 4r_4$$

$$r_2 \rightarrow r_2 + r_4$$

$$r_3 \rightarrow r_3 + \frac{1}{2} r_4.$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 55/2 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 45/2 \end{bmatrix}$$

$$r_1 \rightarrow r_1 - r_3.$$

Thus

$$\begin{array}{l} T_1 = 20 \\ T_2 = 55/2 \\ T_3 = 30 \\ T_4 = 45/2 \end{array}$$