

Math 225, Spring 2010: Midterm 2 Summary

This is a list of the main ideas and results in each section of the book.

Chapter 1: Linear Equations in Linear Algebra

1.1 Systems of Linear Equations

- Matrix of coefficients; augmented matrix of coefficients
- Elementary row operations

1.2 Row Reduction and Echelon Forms

- echelon form
- pivots
- row reduction algorithm

1.3 Vector Equations

- linear combinations of vectors
- vector equation corresponding to $Ax=b$
- $\text{Span}\{v_1, v_2, \dots, v_n\}$

1.4 The Matrix Equation $Ax=b$

- Ax as linear combination of columns of A
- Theorem 1.4: T.F.A.E.:
 - $Ax=b$ has a solution for all b
 - Every b is a linear combination of the columns of A
 - The columns of A span \mathbb{R}^m
 - A has a pivot in every row
- Properties of matrix multiplication

1.5 Solution Sets of Linear Systems

- For $Ax=0$
- For $Ax=b$

1.6 Applications

- In class we only covered chemical equations

1.7 Linear Independence

- Definition of linear **dependence** and linear **independence**
- Meaning of linear dependence for a set of two vectors
- Relation with existence of solutions for $Ax=0$
- Sets of p vector in \mathbb{R}^n if $p>n$

Chapter 2: Matrix Algebra

2.2 Inverses

- Properties of inverses (Theorem 6)
- Elementary matrices
- Algorithm for computing inverses

2.3 Characterization of invertibility in terms of:

- reduced echelon form, number of pivots, null space, linear independence of columns, column space

Chapter 3: Determinants

3.1 Introduction

- Cofactor expansion
- Determinant of a triangular matrix

3.2 Properties of $\det(A)$

- Behavior under row operations
- Relation to invertibility
- Behavior under taking transpose
- Multiplicative properties

3.3 Cramer's Rule, volume, and linear transformations

- Formula for solutions to $Ax=b$
- Formula for inverses
- Behavior under taking transpose
- Multiplicative properties
- Area and volume formulae
- Determinant as distortion factor for linear transformation

Chapter 4: Vector spaces

4.1 Vector spaces and subspaces

- Definitions
- $\text{Span}\{v_1, v_2, \dots, v_n\}$

4.2 $\text{Nul}(A)$ and $\text{Col}(A)$

- Definitions and proof that they're subspaces
- Kernel and range for a linear transformation

4.3 Linear independence and bases

- Definitions of **generating set**, **linear independence**, **basis**
- Theorem: If the vectors in a spanning set are not linearly independent then one of them can be removed.
- Bases for $\text{Nul}(A)$ and $\text{Col}(A)$

4.5 Dimension

- Theorem: If a basis for a vector space has n elements then any set of $k > n$ non-zero vectors must be linearly dependent.
- Theorem: Any two bases for a given vector space have the same number of vectors.
- Definition of dimension

- Theorem: If $\dim(V)=n$ then any set of n non-zero vectors which generates V is a basis, and any set of n linearly independent vectors is a basis.
- Theorem: The dimension of a subspace is no bigger than the dimension of the whole vector space.

4.6 Rank

- Definition
- The Rank Theorem: $\text{rank}(A)+\dim \text{Nul}(A)=n$
- $\text{Rank}(A)=\dim \text{Nul}(A)=\dim \text{Col}(A)$
- Theorem: $n \times n$ matrix A is invertible iff $\text{rank}(A)=n$

Chapter 5: Eigenvectors and eigenvalues

5.1 Eigenvectors and eigenvalues

- Definition, i.e. $Av=\lambda v$
- Eigenspace = $\text{Nul}(A-\lambda I)$
- Eigenvalues for a triangular matrix
- Eigenvectors corresponding to distinct eigenvalues are linearly independent

5.2 Characteristic equation

- eigenvalues and the roots of $\det(A-\lambda I)$
- similar matrices have identical characteristic equation

5.3 Diagonalization

- $A = PDP^{-1}$
- A is diagonalizable iff it has n linearly independent eigenvectors. These are the columns of P and their eigenvalues are the diagonal entries in D .
- A is diagonalizable if it has n distinct eigenvalues

Chapter 6: Orthogonality (and least squares)

6.1-6.4

- Inner product, length, and orthogonality
- Orthogonal and orthonormal sets; orthogonal subspaces
- Orthogonal projection onto subspaces
- Gram-Schmidt process
- Least squares solutions to inconsistent systems $Ax=b$