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**Math. 225 - Quiz 9** April 29, 2010

*Use both sides of the paper if necessary*

Let $\vec{e}_1 = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$, $\vec{e}_2 = \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$.

(1) Show that $\{\vec{e}_1, \vec{e}_2\}$ is an orthonormal set and hence a basis for $\mathbb{R}^2$

$$\left.\begin{aligned} e_1 \cdot e_2 &= -\tfrac{2}{5} + \tfrac{2}{5} = 0 \\ e_1 \cdot e_1 &= \tfrac{1}{5} + \tfrac{4}{5} = 1 \\ e_2 \cdot e_2 &= \tfrac{4}{5} + \tfrac{1}{5} = 1 \end{aligned}\right\}$$ Hence, $\{e_1, e_2\}$ is an orthonormal set.

Also: Orthonormal $\Rightarrow$ linearly independent and $\dim \mathbb{R}^2 = 2$

Hence $\{e_1, e_2\}$ is a basis for $\mathbb{R}^2$.

(2) If $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ find coefficients $c_1, c_2$ such that $\vec{v} = c_1\vec{e}_1 + c_2\vec{e}_2$.

Use $\boxed{c_i = v \cdot e_i}$ Hence $c_1 = v \cdot e_1 = \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{3}{\sqrt{5}}$

$c_2 = v \cdot e_2 = -\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = -\frac{1}{\sqrt{5}}$

Thus $\boxed{\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{3}{\sqrt{5}} e_1 - \frac{1}{\sqrt{5}} e_2.}$