

Use both sides of the paper if necessary

(1) Solve the initial value problem:

$$y \frac{dy}{dx} = x^2, \quad y(0) = 1.$$

(2) Find the general solution to:

$$y' + \frac{2}{x}y = \frac{\sin(x)}{x^2}.$$

①

The equation is separable:

$$y dy = x^2 dx$$

$$\Rightarrow \frac{1}{2} y^2 = \frac{1}{3} x^3 + C$$

If $y(0) = 1$ then

$$\frac{1}{2} \cdot 1 = C$$

$$\text{i.e. } C = \frac{1}{2}$$

The solution is thus

$$y^2 = \frac{2}{3} x^3 + 1$$

To get $y(0) = 1$ we must take

$$y = +\sqrt{\frac{2}{3} x^3 + 1}$$

②

(1st order linear).

Integrating factor:

$$e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$$

Thus we get

$$x^2 y' + 2xy = \sin x$$

$$\Rightarrow \frac{d}{dx}(x^2 y) = \sin x$$

$$\Rightarrow x^2 y = -\cos x + C$$

$$\Rightarrow y = -\frac{\cos x}{x^2} + \frac{C}{x^2}$$