Math. 285 - Quiz 5 April 15, 2004

Use both sides of the paper if necessary

(1) Let $g(x)$ be a periodic function with period 6.

(a) If $g(0) = 5$, what is $g(18)$?

(b) Find $\int_1^7 g(x)dx$ if $\int_{-3}^3 g(x)dx = 11$.

(2) Let $f(x)$ be the periodic extension of the function defined by

$$f(x) = \begin{cases} +1 & if \quad 0 < x < 1 \\ -1 & if \quad -1 < x < 0 \end{cases}$$

(a) Complete the definition by picking values for $f(0)$ and $f(1)$ so that the Fourier series for f converges to $f(x)$ at all x.

(b) Compute the Fourier series.

(1) (a) $g(x+6) = g(x)$

$\Rightarrow g(18) = g(0+3\cdot 6) = g(0) = \underline{\underline{5}}$

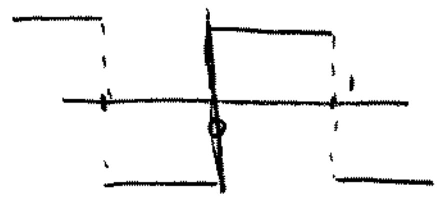
(b) $\int_a^{a+6} g(x)dx = \int_{-3}^{3} g(x)dx$ for any a

Hence, taking $a=1$,

$\int_1^7 g(x)dx = \int_{-3}^{3} g(x)dx = \underline{\underline{11}}$

2 (a) The Fourier series converges to the <u>average</u>

$$\bar{f}(x) = \frac{f^+(x) + f^-(x)}{2}$$



where $f^{\pm}(x) = \lim_{t \to x^{\pm}} f(t)$.

But $\bar{f}(1) = \frac{-1+1}{2} = 0$

$\bar{f}(0) = \frac{1+-1}{2} = 0$

Hence we should <u>define</u> $f(0) = 0 = f(1)$

(b) $f(x)$ is <u>odd</u>, so

$$b_n = \frac{1}{1} \int_{-1}^{1} f(x) \cos \frac{n\pi}{1} x \, dx = 0 \quad \forall n$$

$$a_n = \int_{-1}^{1} f(x) \sin n\pi x \, dx \qquad n = 1, \ldots$$

$$= \int_0^1 \sin n\pi x \, dx - \int_{-1}^{0} \sin n\pi x \, dx$$

$$= \left. \frac{\cos n\pi x}{n\pi} \right|_1^0 - \left. \frac{\cos n\pi x}{n\pi} \right|_0^{-1}$$

$$= \frac{1}{n\pi} \left(1 - (-1)^n - (-1)^n + 1 \right) = \begin{cases} 0 & n \text{ even} \\ \frac{4}{n\pi} & n \text{ odd} \end{cases}$$

Also, $a_0 = \int_{-1}^{1} f(x) \, dx = 0$

Hence $$\boxed{f(x) = \sum_{n \text{ odd}} \frac{4}{n\pi} \sin n\pi x}$$