(c) Show that there are no asymptotic directions at a point where the Gauss curvature is positive.

(5)

$\vec{u}$ asymptotic $\Longleftrightarrow$ $k(\vec{u}) = 0$

If $k_1 \geqslant k_2$ (where k_1, k_2 are the principal curvatures).

then $\boxed{k_2 \leq 0 \leq k_1}$ ~~at an asymptotic~~ ——— (*)

if $\exists$ an asymptotic direction.

But $K > 0 \Longleftrightarrow$ k_1 and k_2 have the same sign (and are non zero).

——— (**).

Since (*) and (**) are contradictory,

$K > 0 \Longrightarrow$ no asymptotic points!

Question 6

Let M be a generalized cylinder, i.e. a ruled surface parameterized by

$$\sigma(u,v) = \vec{\beta}(u) + v\vec{\delta}$$

where $\vec{\beta}(u)$ is the base curve and $\vec{\delta}$ is a fixed direction vector. Show that M is flat.

(15)

Compute:

$$\frac{\partial \sigma}{\partial u} = \vec{\beta}'(u)$$

$$\frac{\partial \sigma}{\partial v} = \vec{\delta}$$

Hence

$$\frac{\partial^2 \sigma}{\partial u^2} = \vec{\beta}''(u)$$

$$\frac{\partial^2 \sigma}{\partial u \partial v} = 0 \implies \boxed{M = 0}$$

$$\frac{\partial^2 \sigma}{\partial v^2} = 0 \implies \boxed{N = 0}$$

Thus $K = \dfrac{LN - M^2}{EG - F^2} = 0$!