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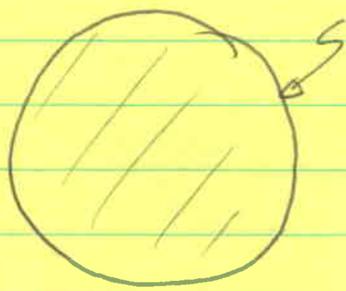
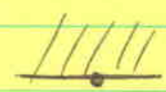
1 (a)

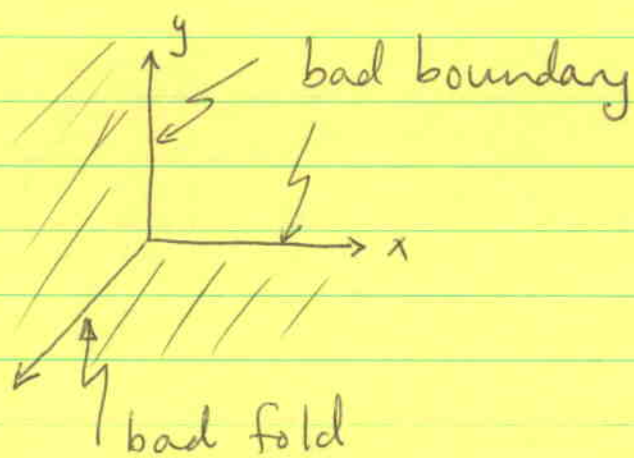
bad! any neighbourhood looks like this:

(b)



any point on the boundary looks like a point on the boundary of a half-plane , i.e. does not live in a nbd which looks like an open set in $\mathbb{R}^2$.

(c)



(4)

(a) $\sigma(u,v) = (u, uv, v)$ is clearly 1-1

$$\sigma_* = \begin{bmatrix} 1 & 0 \\ v & u \\ 0 & 1 \end{bmatrix}$$ which is clearly 1-1

Good patch

(b) $\sigma(u,v) = (u^2, u^3, v)$ is 1-1 :

~~$\sigma(u_1,v_1) = \sigma(u_2,v_2)$~~

$$\Rightarrow \begin{cases} u_1^3 = u_2^3 \\ v_1 = v_2 \end{cases} \Rightarrow \begin{cases} u_1 = u_2 \\ v_1 = v_2 \end{cases}$$

$$\sigma_* = \begin{bmatrix} 2u & 0 \\ 3u & 0 \\ 0 & 1 \end{bmatrix}$$ which is 1-1 <u>except</u> if $u=0$

Good patch unless domain includes $u=0$

(c) $\sigma(u,v) = (u, u^2, v+v^3)$ <u>is</u> 1-1 :

The tricky part is to verify that

$v_1 + v_1^3 = v_2 + v_2^3 \Rightarrow v_1 = v_2$.

But $v_1 + v_1^3 = v_2 + v_2^3 \Rightarrow (v_1 - v_2)(1 + v_1^2 + v_1 v_2 + v_2^2) = 0$

Since $1+v_1^2+v_1v_2+v_2^2$ is never zero (check!).
this means $v_1 = v_2$.

$$\sigma_* = \begin{bmatrix} 1 & 0 \\ 2u & 0 \\ 0 & 1+3v^2 \end{bmatrix}$$, which is always 1-1

Good Patch.

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$$\sigma(u,v) = (u+v,\ u-v,\ uv) \quad ; \quad (u,v) \in \mathbb{R}^2$$

(i) σ is 1-1 : $\sigma(u_1,v_1) = \sigma(u_2,v_2)$

$$\Longrightarrow \begin{cases} u_1+v_1 = u_2-v_2 \\ u_1-v_1 = u_2-u_2 \end{cases}$$

$$\Longrightarrow \begin{cases} u_1=u_2 \\ v_1=v_2 \end{cases}$$ (check the linear algebra!)

(ii) σ_* is 1-1 : $\sigma_* = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ v & u \end{bmatrix}$ which is 1-1 for all (u,v).

(iii) image = M : If $(x,y,z) = \sigma(u,v)$

$$= (u+v,\ u-v,\ uv)$$

then $\dfrac{x^2-y^2}{4} = \dfrac{(u+v)^2-(u-v)^2}{4}$

$$= uv$$

$$= z \quad \text{i.e. } \sigma(u,v) \in M$$

Conversely, if $\dfrac{x^2-y^2}{4} = z$ we can solve for (u,v)

i.e. take $\boxed{u = \dfrac{x+y}{2}\ ,\ v = \dfrac{x-y}{2}}$

i.e. $(x,y,z) = \sigma\left(\dfrac{x+y}{2}, \dfrac{x-y}{2}\right)$

Hence any ~~every~~ $(x,y,z) \in M$ is in the image of σ.