HOMEWORK 9

p288 #10

If $F(\sigma(u,v)) = \bar{\sigma}(f(u), g(v))$ —— $(*)$

(a) Then $F(\sigma(t, v_0)) = \bar{\sigma}(f(t), g(v_0))$

$$= \bar{\sigma}(s, g(v_0)) \quad ; \quad s = f(t)$$

ie parameter curve $\sigma(t, v_0)$ through $\vec{p} = \sigma(0, v_0)$ gets mapped to parameter curve $\bar{\sigma}(s, g(v_0))$ through $F(\vec{p}) = \bar{\sigma}(f(0), g(v_0))$, with the new curve parameterized by $s = f(t)$. Similarly, the parameter curve $\sigma(u_0, t)$ gets mapped to parameter curve

$\bar{\sigma}(f(u_0), s)$ with $s = g(t)$

(b) $(*) \Rightarrow F_*\left(\frac{\partial \vec{\sigma}}{\partial u}\right) = \frac{df}{du}\left(\frac{\partial \vec{\bar{\sigma}}}{\partial x}\right)$ (using x, y for the coordinates on N)

and $F_*\left(\frac{\partial \vec{\sigma}}{\partial v}\right) = \left(\frac{dg}{dv}\right)\left(\frac{\partial \vec{\bar{\sigma}}}{\partial v}\right)$

Hence $\left|F_*\left(\frac{\partial \vec{\sigma}}{\partial u}\right)\right| = \left|\frac{df}{du}\right|^2 \bar{E}$ etc.

The result follows from this. (details omitted)