Math 510 - Fall 2004
Algebraic Curves and Riemann Surfaces
Problem Set 4
Due Date: Tuesday December 5

Question 1

From Griffiths:

- Ex 1.1 (p56),
- Ex 7.1 (p83),
- Ex 7.4 (p86),
- Ex 7.5 (p89)
- (Extra credit: Ex 7.2 or 7.3 (p85))

Question 2

(a) Let C_1 and C_2 be curves in $\mathbb{P}^2$ and let p be a point in $C_1 \cap C_2$. Prove that $(C_1 \cdot C_2)_p = 1$ if and only if p is a smooth point of both curves and the tangent lines to C_1 and C_2 at p are distinct.

(b) Suppose that C_1 and C_2 have degree n and m respectively. Show that if every point of intersection is a smooth point of both curves, and if the tangent lines are distinct at all points of intersection, then the intersection $C_1 \cap C_2$ consists of nm distinct points.

Question 3

Show that given any 5 points in $\mathbb{P}^2$ there is at least one conic containing them. Deduce that a projective curve C of degree 4 in $\mathbb{P}^2$ with 4 singular points is reducible. [Hint: show that any conic containing the 4 singular points and one other point of C must share a component with C.]

Question 4

Prove Pappus' Theorem: Let L and M be lines in $\mathbb{P}^2$. Let p_1, p_2, p_3 be 3 points on L but not on M, and let q_1, q_2, q_3 be 3 points on M but not on L. Let L_{ij} be the line through p_i and q_j. Prove that the 3 points of intersection of the lines L_{ij} and L_{ji} are collinear. [Hint: The proof is similar to the proof of Pascal's Theorem, but with the conic being $L + M$, i.e. with a reducible conic]