MATH 231 U1, Spring 2009
Answers to HW 11, Section 7.1
Due Monday February 16th, 2009

#4. Find the solution of the given differential equation satisfying the initial value condition.

$$y' = -2y \; ; \quad y(0) = -6$$

Answer:
We know that the general solution for an exponential differential equation $y' = ky$ has form:

$$y(t) = Ae^{kt}$$

And $k = -2$ in this problem. So, this gives us

$$y(t) = Ae^{-2t}$$

To find A we plug in 0 and get

$$-6 = y(0) = Ae^0 = A$$

So our answer is

$$y(t) = -6e^{-2t}$$

#14. A bacterial culture grows exponentially with a growth constant of 0.44 hour^{-1}. Find its doubling time.

Answer: The growth constant $k = 0.44$ so we know if $y(t) =$ number of bacteria at time t, and A=number of bacteria we start with= $y(0)$ then

$$y(t) = Ae^{0.44t}$$

To find the doubling time, we solve for the t which gives us $y(t) = 2A$.

$$2A = y(t) = Ae^{0.44t}$$

$$2 = e^{0.44t}$$

$$\ln 2 = 0.44t$$

$$\frac{\ln 2}{0.44} = t$$

is the doubling time in hours.

#20. The radioactive element cesium-137 has a decay constant of -1.3863 day^{-1}. Find its half life.

Answer:
The decay constant is $k = -1.3863$, so we know that

$$y(t) = Ae^{-1.3863t}$$

To find the half life, solve for the t which gives $y(t) = \frac{A}{2}$

$$\frac{A}{2} = y(t) = Ae^{-1.3863t}$$

$$0.5 = e^{-1.3863t}$$

$$\ln 0.5 = -1.3863t$$

$$\frac{\ln 0.5}{-1.3863} = t$$

So, the half life is $\frac{\ln 0.5}{-1.3863}$ days.

#56. It is reported that Prozac has a half-life 2 to 3 days but may be found in your system for several weeks after you stop taking it. What percentage of the original dosage would remain after 2 weeks if the half life is 2 days? How much would remain if the half-ife is 3 days?

Answer:
Assuming a half-life of 2 days, let $y(t)$ be the function that tells you the percentage of your initial dose which is left in your blood stream after t days.

$y(t) = 100e^{kt}$ (for a negative k since this is exponential decay).

Now we must find the decay constant k. We know that

$$y(2) = 50 = 100e^{k2}$$

So,

$$0.5 = e^{k2}$$

$$\ln 0.5 = k2$$

$$\frac{\ln 0.5}{2} = k$$

So our function is

$$y(t) = 100e^{\frac{\ln 0.5}{2}t}$$

To answer the question, we find $y(14) = 100e^{\frac{14 \ln 0.5}{2}} \approx 0.78$ percent.

Repeat with a half life of 3 days instead of 2 and compare.

#63. If you win a "million dollar" lottery, would you be better off getting your money in four annual installments of $\$280,000$ or in one lump sum of $\$1$ million? Assume 8% interest and payments made at the beginning of the year.

Answer:
Here notice that they are NOT compounding continuously, but only once a year. (Note: my answer in class was assuming 4 years of accumulation, in the book, they stop counting after 3 years of accumulation at the point when you get your 4th installment. I also said that without a calculator I was guessing that you should take the 1,000,000, but the actual numbers have proved me wrong.)

At the time when you would get your 4th payment, the 1,000,000 would have been accruing interest for 3 years. So with the one time payment, you would have

$$1,000,000(1.08)^3 = 1259712$$

With the 4 installments of $280,000$ you have

$$280000 + 280000(1.08) + 280000(1.08)^2 + 280000(1.08)^3 = 1261711$$

So, you should take the 4 installments.