MATH 231 U1, Spring 2009
Answers to HW 2, Section 6.2 problems 10, 16, 37, 38, 45
Monday January 26th, 2009

6.2 #10:
$\int e^{2x} \cos x \; dx$

Using integration by parts with $u = e^{2x}$ and $dv = \cos x \, dx$ we have $du = 2e^{2x} \, dx$ and $v = \sin x$

$$\int e^{2x} \cos x \; dx = e^{2x} \sin x - \int 2e^{2x} \sin x \; dx$$

Using integration by parts with $u = e^{2x}$ and $dv = \sin x \, dx$ we have $du = 2e^{2x} \, dx$ and $v = -\cos x$

$$\int e^{2x} \cos x \; dx = e^{2x} \sin x - 2 \int e^{2x} \sin x \; dx = e^{2x} \sin x - 2\Big(- e^{2x} \cos x + \int 2e^{2x} \cos x \, dx\Big)$$

So, we have

$$\int e^{2x} \cos x \; dx = e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x \, dx$$

Thus,

$$5 \int e^{2x} \cos x \; dx = e^{2x} \sin x + 2e^{2x} \cos x$$

$$\int e^{2x} \cos x \; dx = \frac{1}{5}\big(e^{2x} \sin x + 2e^{2x} \cos x\big) + C$$

#16:
$\int x^2 e^{3x} \; dx$

Using integration by parts with $u = x^2$ and $dv = e^{3x} \, dx$ we have $du = 2x \, dx$ and $v = \frac{e^{3x}}{3}$.

$$\begin{aligned} \int x^2 e^{3x} \; dx &= \frac{1}{3} x^2 e^{3x} - \frac{2}{3} \int x e^{3x} \, dx \\ &= \frac{1}{3} x^2 e^{3x} - \frac{2}{3}(\frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} \, dx) \\ &= \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C \end{aligned}$$

The second to the third step is another application if IBP, with $u = x$ and $dv = e^{3x} \, dx$.

#37:
How many times would integration by parts need to be performed to evalulate $\int x^n \sin x \, dx$ (where n is a positive integer)?

n times. Letting $u = x^n$, each time you do integration by parts the exponent on n goes down by 1.

#38:
How many times would integration by parts need to be performed to evalulate $\int x^n \ln x \, dx$ (where n is a positive integer)?

Once. Try it!

#45:
$\int_0^1 x^4 e^x \; dx$
Using the reduction formula $\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx$

$$\begin{aligned}
\int x^4 e^x \; dx &= x^4 e^x - 4 \int x^3 e^x \; dx \\
&= x^4 e^x - 4(x^3 e^x - 3 \int x^2 e^x \; dx) \\
&= x^4 e^x - 4x^3 e^x + 12(x^2 e^x - 2 \int x e^x \; dx) \\
&= x^4 e^x - 4x^3 e^x + 12x^2 e^x + 24(x e^x - \int e^x \; dx) \\
&= x^4 e^x - 4x^3 e^x + 12x^2 e^x + 24x e^x - 24 e^x
\end{aligned}$$

Now evaluate $x^4 e^x - 4x^3 e^x + 12x^2 e^x + 24x e^x - 24 e^x |_0^1 = 9e - 24$

(Remember, using a reduction formula is just doing integration by parts, but where someone else has already done the work of choosing u and dv and plugging into $uv - \int v \, du$ for you.)