9.1 Worksheet Answers 2(c), 4(b), 5

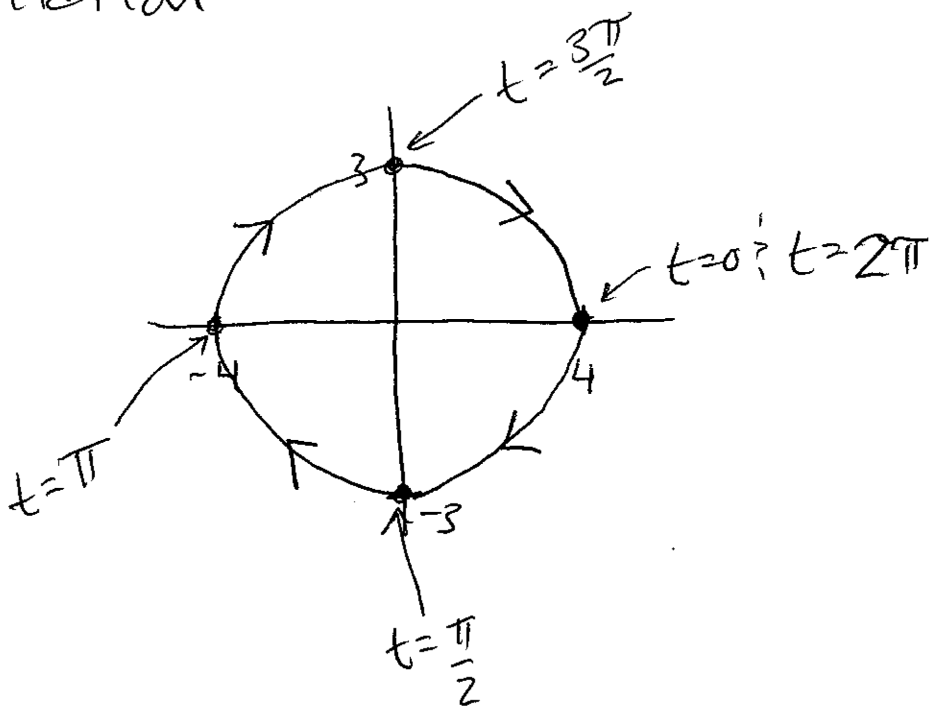
2(c). Sketch $\begin{cases} x = 4\cos t \\ y = -3\sin t \end{cases}$ for $0 \leq t \leq 2\pi$

and indicate the orientation.

This will be an <u>ellipse</u>, and by plugging in $t=0$, $t=\frac{\pi}{2}$, $t=\frac{3\pi}{2}$, $t=\pi$

we can get an idea of where it lies and the orientation

t	(x, y)
0	$(4, 0)$
$\frac{\pi}{2}$	$(0, -3)$
π	$(-4, 0)$
$\frac{3\pi}{2}$	$(0, 3)$



4(b). Construct 2 <u>different</u> sets of parametric equations for the line seg. between $(1,3)$ and $(5,15)$

4(b).

1 For t in $[0,1]$

$x(t) = 1 + 4t$

$y(t) = 3 + 12t$

(set $x = a + bt$, $y = c + dt$, plug in $t=0$ and $t=1$ and solve)

2 For t in $[0,3]$

$x(t) = 1 + \frac{4}{3}t$

$y(t) = 3 + 4t$

(to do this from scratch, set $x = a + bt$, $y = c + dt$, plug in $t=0$ and $t=3$ and solve)

3 (Bonus) For t in $[0,1]$

$x(t) = 1 + 4t^2$

$y(t) = 3 + 12t^2$

Here the rate at which we move along the line segment as t goes from 0 to 1 will be different than in 1

5. For $\begin{cases} x_1(t) \\ y_1(t) \end{cases}$ we get $x_1 + 1 = t$ so

$y_1(x_1) = (x_1+1)^2 - 2(x_1+1)$ where x_1 can be any real number since $x_1 + 1 = t$ and t is in $(-\infty, \infty)$.

5. For $\begin{cases} x_2(t) \\ y_2(t) \end{cases}$ we get $x_2 + 1 = t^2$

so $\pm\sqrt{x_2+1} = t$

$y_2(t) = (t^2)^2 - 2t^2$

so $y_2(x_2) = (x_2+1)^2 - 2(x_2+1)$

where x_2 has domain $[-1, \infty)$, because

$x_2 = t^2 - 1$, and $t^2 \geq 0$

so we get the same curve as for x_1, y_1 except we only get it on the domain $x_2 \in [-1, \infty)$.

for $\begin{cases} x_3(t) \\ y_3(t) \end{cases}$ we get $x_3 + 1 = t^3$

$\sqrt[3]{x_3+1} = t$

So

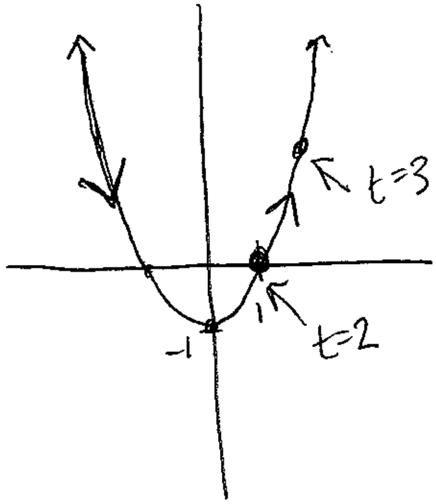
$y_3(t) = (t^3)^2 - 2t^3$

$y_3(x_3) = (x_3+1)^2 - 2(x_3+1)$

and x_3 can be any real # again $\left(\text{as for } \begin{cases} x_1(t) \\ y_1(t) \end{cases}\right)$

because $x_3 = t^3 - 1$ and t^3 can be any real number.

(1)



$\begin{cases} x_1(t) \\ y_1(t) \end{cases}$ for t in $(-\infty, \infty)$

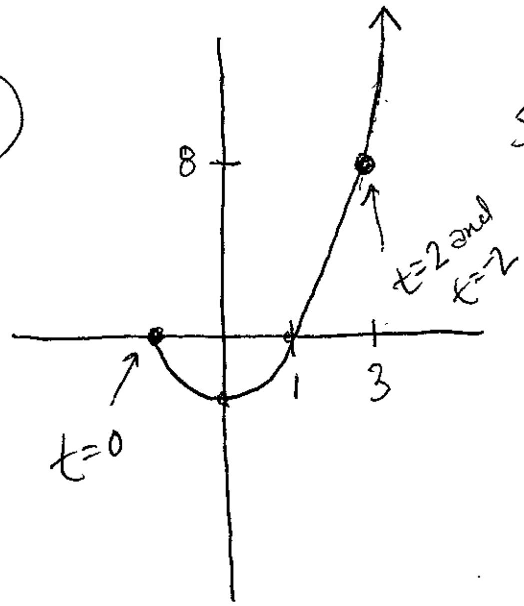
parabola $(x+1)^2 - 2(x+1)$

for x in $(-\infty, \infty)$

$t=2$ gives us point $(1,0)$

$t=3$ gives us point $(2,3)$

(2)



$\begin{cases} x_2(t) \\ y_2(t) \end{cases}$ for t in $(-\infty, \infty)$

Same parabola, only graphed

for x in $[-1, \infty)$

$t=2$ gives us point $(3,8)$

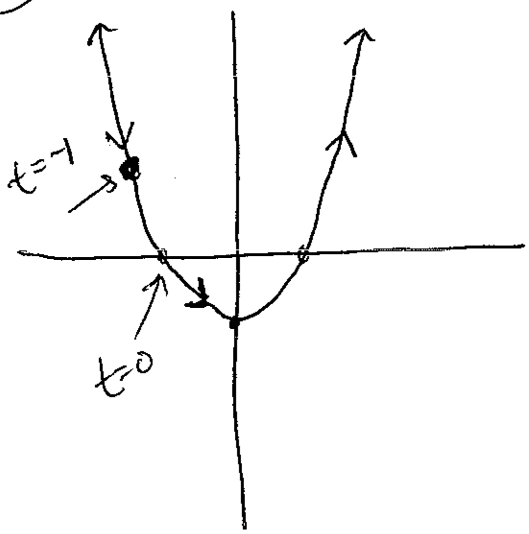
$t=3$ gives us point $(8,48)$

$t=0$ gives us point $(-1,0)$

$t=-2$ gives us point $(3,8)$

$t=-3$ gives us point $(8,48)$

(3)



$\begin{cases} x_3(t) \\ y_3(t) \end{cases}$ for t in $(-\infty, \infty)$

same shape as (1), but

note:

$t=2$ gives us point $(7,35)$

$t=3$ gives us point $(26,675)$

$t=-1$ gives us point $(-2,3)$

$t=0$ gives us point $(-1,0)$