§9.3 #8, 24, 30

8. Find Arclength of $x = t^2 \cos t$, $y = t^2 \sin t$ $\quad -1 \le t \le 1$

$x'(t) = -t^2 \sin t + 2t\cos t$

$y'(t) = t^2 \cos t + 2t \sin t$ $\quad$ so

$$\sqrt{[x'(t)]^2 + [y'(t)]} = \sqrt{t^4\sin^2 t - \cancel{4t^3 \sin t \cos t} + 4t^2\cos^2 t + t^4\cos^2 t + \cancel{4t^3 \sin t \cos t} + 4t^2 \sin^2 t}$$

$$= \sqrt{t^4(\sin^2 t + \cos^2 t) + 4t^2(\cos^2 t + \sin^2 t)} = \sqrt{t^4 + 4t}$$

$$= \sqrt{t^2}\sqrt{t^2+4}$$

$$= |t|\sqrt{t^2+4}$$

So arc length is

$$S = \int_{-1}^{1} |t|\sqrt{t^2+4}\,dt = \int_{-1}^{0} -t\sqrt{t^2+4}\,dt + \int_0^1 t\sqrt{t^2+4}\,dt$$

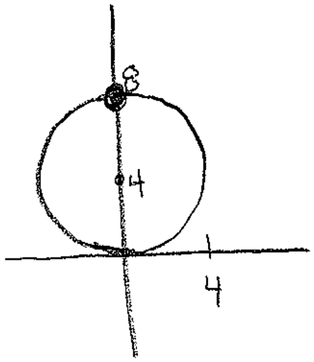
$u = t^2+4$

$du = 2t\,dt$

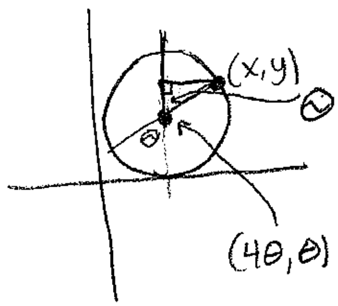
$$= \frac{-1}{2}\int_5^4 \sqrt{u}\,du + \frac{1}{2}\int_4^5 \sqrt{u}\,du = \int_4^5 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_4^5$$

$$= \frac{2}{3}\left(5^{3/2} - 4^{3/2}\right) = \frac{2}{3}\left(5^{3/2} - 8\right)$$

30. Find parametric equations for the cycloid traced by the circle of radius 4, where the "special point" starts at (0,8). Then find the arc length of one arch.



The __center__ will be at $(4\theta, 4)$, where θ is the # of radians the circle has rolled

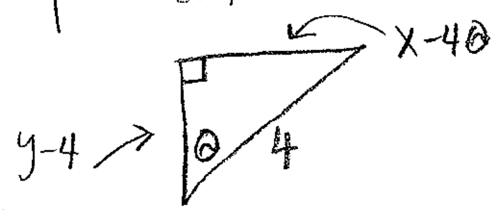


$\sin\theta = \dfrac{x-4\theta}{4}$ so $x = 4\theta + 4\sin\theta$

$\cos\theta = \dfrac{y-4}{4}$ so $y = 4 + 4\cos\theta$

ANSWER

(OUR PARAMETER IS θ)



Arc length: $x'(\theta) = 4 + 4\cos\theta$, $y'(\theta) = -4\sin\theta$

$(x')^2 + (y')^2 = 16 + 32\cos\theta + 16\cos^2\theta + 16\sin^2\theta$

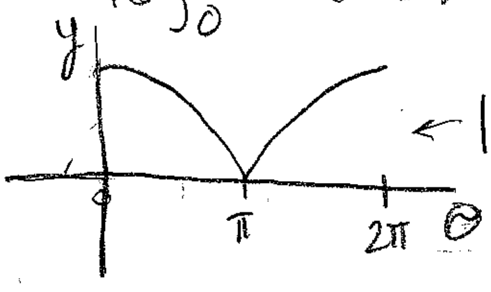
$= 32 + 32\cos\theta$

$= 32(1+\cos\theta)$ 1/2 ang formula

$= 64\cos^2(\theta/2)$

$$S = \int_0^{2\pi} \sqrt{64\cos^2(\theta/2)}\, d\theta = \int_0^{2\pi} 8\left|\cos(\theta/2)\right| d\theta$$

$$= 16\int_0^{\pi} \cos(\theta/2) = 32\sin(\theta/2)\Big|_0^{\pi} = 32$$



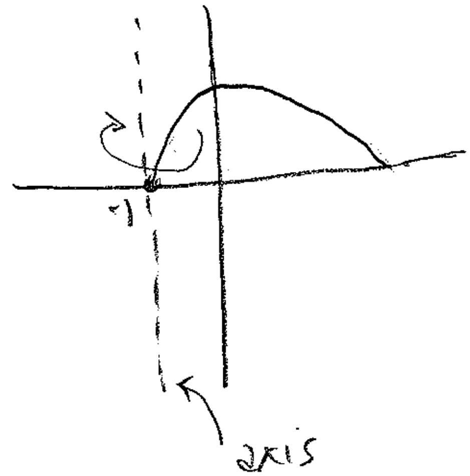
24. Find surface area of solid of revolution for

$x = t^2 - 1$

$-2 \le t \le 0$

$y = t^3 - 4t$

revolved about $x = -1$



$x'(t) = 2t$

$y'(t) = 3t^2 - 4$

$$SA = \int_{-2}^{0} 2\pi |t^2| \sqrt{4t^2 + (3t-4)^2}\, dt$$

(you could perhaps find a way to evaluate this, you would get

≈ 83.923)