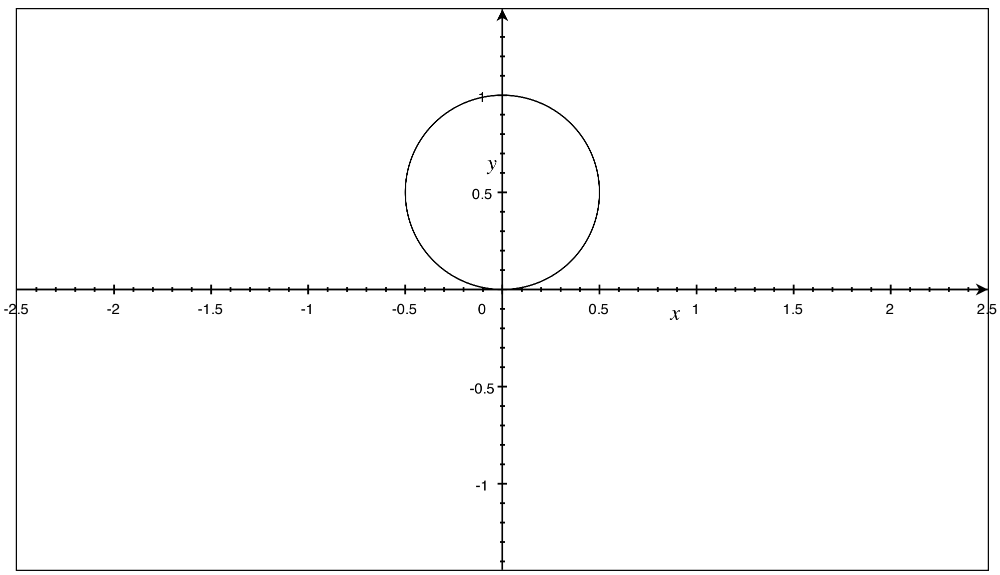
MATH 231 U1, Spring 2009
HW 31 (worksheet) Answers, Section 9.4/9.5
Wednesday, April 29th, 2009

#3. Use the previous problem to help graph these polar functions.
(a) $r = \sin\theta$



$$\theta = 0 \qquad r = 0$$
$$\theta = \frac{\pi}{4} \qquad r = \frac{\sqrt{2}}{2}$$
$$\theta = \frac{\pi}{2} \qquad r = 1$$
$$\theta = \frac{3\pi}{4} \qquad r = \frac{\sqrt{2}}{2}$$
$$\theta = \pi \qquad r = -1$$

So, the curve gets traced once for $\theta \in [0, \pi]$.

#6. Find a polar equation corresponding to the given rectangular equation.
(b) $y^2 - x^2 = 1$.

We know that $x = r\cos\theta$ and $y = r\sin\theta$, so we get

$$r^2 \sin^2\theta - r^2 \cos^2\theta = 1$$

and this is a polar equation which corresponds to the rectangular equation.
If we simplify we get:

$$\begin{aligned} y^2 - x^2 &= 1 \\ r^2 \sin^2\theta - r^2\cos^2\theta &= 1 \\ r^2(\sin^2\theta - \cos^2\theta) &= 1 \\ r^2(-\cos(2\theta)) &= 1 \\ r^2 &= \frac{-1}{\cos(2\theta)} \\ r &= \pm\sqrt{-\sec(2\theta)} \end{aligned}$$

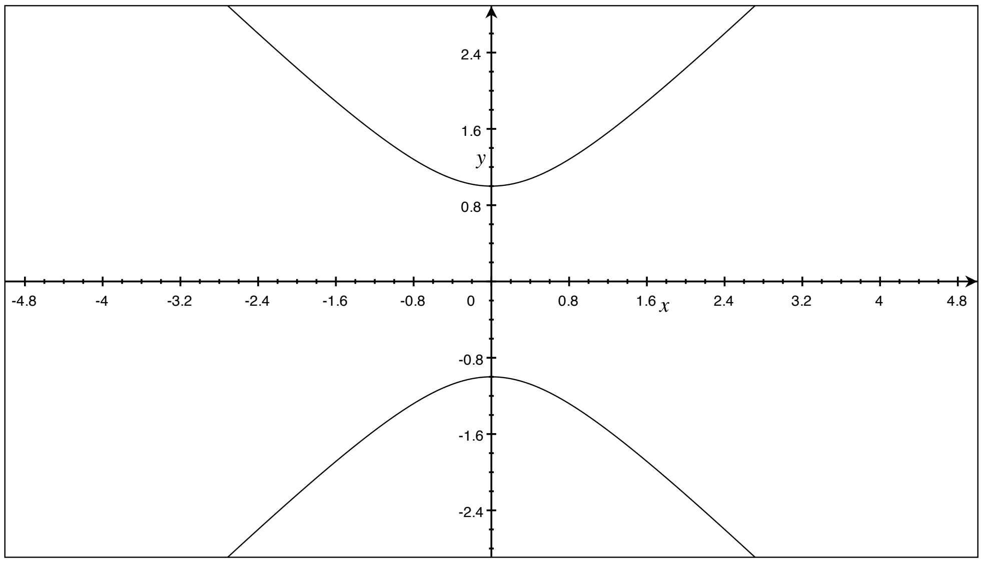
Notice here that because we are taking the square root $\sqrt{-\sec(2\theta)}$, we will get a restricted domain for θ. There will not be points on our curve which correspond to θ which makes $\sec(2\theta) > 0$, because then $-\sec(2\theta) < 0$ and the square root DNE. Therefore, the range which graphs this hyperbola will be $[\frac{\pi}{4}, \frac{\pi}{2}] \cup [\frac{5\pi}{4}, \frac{7\pi}{4}]$

Also, including both the positive and negative root in THIS particular case does not matter. If you graph $r = \sqrt{-\sec(2\theta)}$, you will get the same graph as $r = -\sqrt{-\sec(2\theta)}$ (although one will trace the top part first, and the other will trace the bottom part first). So, when we include both we get the graph traced twice. For SOME hyperbolas, (see the example in the book) one half of the hyperbola corresponds to the positive root, and the other half corresponds to the negative root, but this is not the case in this problem.

So, to get the graph traced once, we would plot, for example,

$$r = \sqrt{-\sec(2\theta)}$$

for $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}] \cup [\frac{5\pi}{4}, \frac{7\pi}{4}]$



#7. For the given curves, give a Polar and Cartesian equation for each.
(e) A circle of radius 4 centered about the point $(3, 4)$.

For this problem, it is easier to start with the Cartesian equation! It will be

$$(x - 3)^2 + (y - 4)^2 = 16$$

.

If we use the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we get the equation

$$(r\cos\theta - 3)^2 + (r\sin\theta - 4)^2 = 16$$

and we can STOP there.

Simplifying this equation would yield:

$$r^2 - 10r\cos(\theta - \arctan(4/3)) + 25 = 16$$

Because generally:
The polar equation of a circle with center with polar coordinates: (c, α) and radius a:

$$r^2 - 2cr\cos(\theta - \alpha) + c^2 = a^2$$

(You need NOT memorize this.)

The moral of the story: depending on the problem, Cartesian coordinates may be better, or Polar coordinates may be better. So its good to have both!