MATH 231 U1, Spring 2009
HW 32 (9.5) Answers
Due Monday, May 4th, 2009

#4. Find the slope of the tangent line to the polar curve at the given point:
$r = \cos 2\theta$ at $\theta = \frac{\pi}{4}$.

ANSWER

$$\left.\frac{dy}{dx}\right|_{\theta=a} = \frac{f'(a)\sin a + f(a)\cos a}{f'(a)\cos a - f(a)\sin a}$$

And $r = f(\theta) = \cos 2\theta$, so $f'(\theta) = -2\sin 2\theta$

Using the formula

$$\left.\frac{dy}{dx}\right|_{\theta=a} = \frac{f'(a)\sin a + f(a)\cos a}{f'(a)\cos a - f(a)\sin a}$$

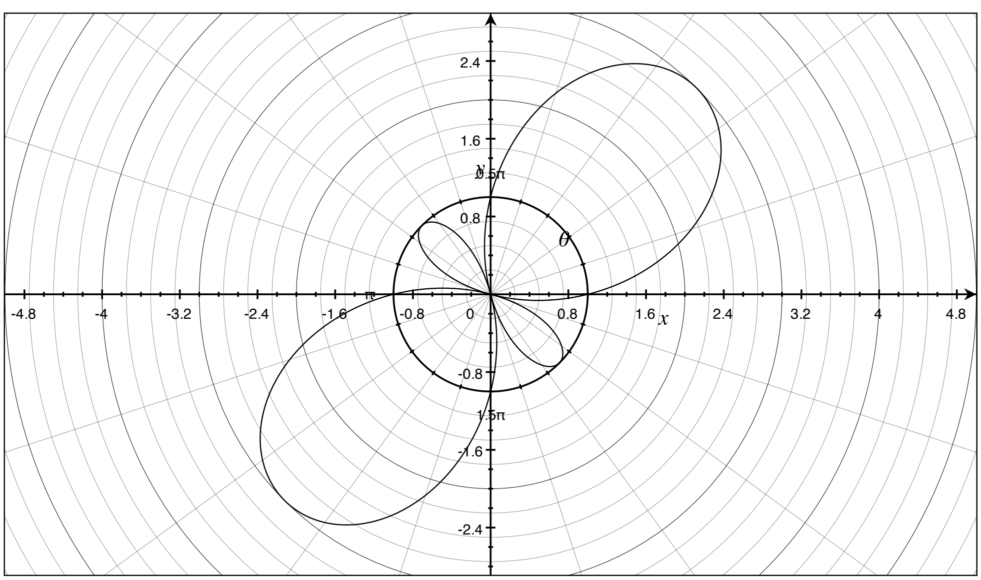
we conclude

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{(-2\sin(2\pi/4))\sin(\pi/4) + \cos(2\pi/4)\cos(\pi/4)}{(-2\sin(2\pi/4))\cos(\pi/4) - \cos(2\pi/4)\sin(\pi/4)} = \frac{(-2)\frac{\sqrt{2}}{2} + 0}{(-2)\frac{\sqrt{2}}{2} - 0} = 1$$

#21. Find the area of a small loop of $r = 1 + 2\sin 2\theta$.

ANSWER

The graph of this equation is:



The lower small loop is graphed for θ between $\frac{7\pi}{12}$ and $\frac{11\pi}{12}$. So its area is given by

$$\begin{aligned}
\int_{7\pi/12}^{11\pi/12} \frac{1}{2}(1+2\sin 2\theta)^2 \, d\theta &= \frac{1}{2}\int_{7\pi/12}^{11\pi/12} (1+4\sin 2\theta + 4\sin^2 2\theta) \, d\theta \\
&= \frac{1}{2}\int_{7\pi/12}^{11\pi/12} (1+4\sin 2\theta + 2(1-\cos 4\theta)) \, d\theta \\
&= \frac{1}{2}\int_{7\pi/12}^{11\pi/12} (3+4\sin 2\theta - 2\cos 4\theta)) \, d\theta \\
&= \frac{1}{2}\left(3\theta - 2\cos 2\theta - \frac{1}{2}\sin 4\theta\right)\Big|_{7\pi/12}^{11\pi/12} \\
&= \frac{\pi}{2} - \frac{3\sqrt{3}}{4}
\end{aligned}$$

#22. Find the area of a large loop of $r = 1 + 2\sin 2\theta$.

ANSWER

The large loop (the rightmost one) is traced out for θ between $\frac{-\pi}{12}$ and $\frac{7\pi}{12}$. So its area is given by

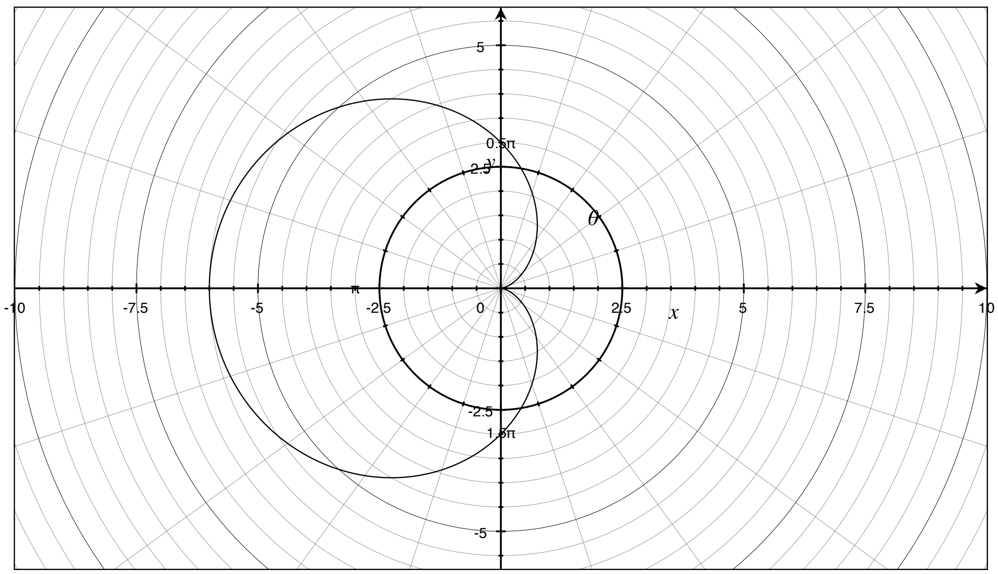
$$\int_{-\pi/12}^{7\pi/12} \frac{1}{2}(1+2\sin 2\theta)^2 \, d\theta$$

which you can easily evaluate, since you already found the antiderivative in # 21. The answer is $\frac{\pi}{2} + \frac{3\sqrt{3}}{4}$.

#36. Write down the integral which gives the arc length of $r = 3 - 3\cos\theta$.

ANSWER

Here is a picture of the curve, which gets traced out as θ increases from 0 to 2π.



So, the arc length is given by

$$s = \int_a^b \sqrt{[f'(\theta)]^2 + [f(\theta)]^2}\ d\theta$$

In this case, $f'(\theta) = 3\sin\theta$, and we get the integral

$$s = \int_0^{2\pi} \sqrt{[3\sin\theta]^2 + [3 - 3\cos\theta]^2}\ d\theta$$