

MATH 231 U1, Spring 2009  
HW 33 (9.6) Answers  
Due Wednesday, May 6th, 2009

#6. Find an equation for the indicated conic section: ellipse with foci  $(1, 2)$  and  $(1, 4)$  and vertices  $(1, 1)$  and  $(1, 5)$ .

ANSWER

The center must be  $(1, 3)$ , so  $x_0 = 1$ ,  $y_0 = 3$ .

We know  $c$  is the distance from the center to to a focus, so  $c = 1$

We know  $a$  is half the distance between the vertices (because  $a$  is the distance from the center to each vertex), so  $a = 2$ .

Since  $a^2 - b^2 = c^2$  we deduce that  $b = \sqrt{3}$ .

So the equation is

$$\frac{(x - 1)^2}{3} + \frac{(y - 3)^2}{4} = 1.$$

#18. Identify the conic section and find each focus and vertex.

$$\frac{(x + 1)^2}{4} - \frac{(y - 3)^2}{4} = 1$$

ANSWER

This is a hyperbola with center  $(-1, 3)$ , so  $x_0 = -1$ ,  $y_0 = 3$ . It is also clear from the equation that  $a = 2$ ,  $b = 2$ , so therefore  $c = \sqrt{a^2 + b^2} = \sqrt{8}$ . So, the foci are:  $(-1 + \sqrt{8}, 3)$  and  $(-1 - \sqrt{8}, 3)$  and the vertices are  $(1, 3)$  and  $(-3, 3)$ .

#28. Graph the conic section and find an equation for:

All points such that the sum of the distances fro the points  $(3, 1)$  and  $(-1, 1)$  equals 6.

ANSWER

This is an ellipse with foci  $(3, 1)$  and  $(-1, 1)$ . So, the center of the ellipse is  $(1, 1)$  (halfway between the foci).

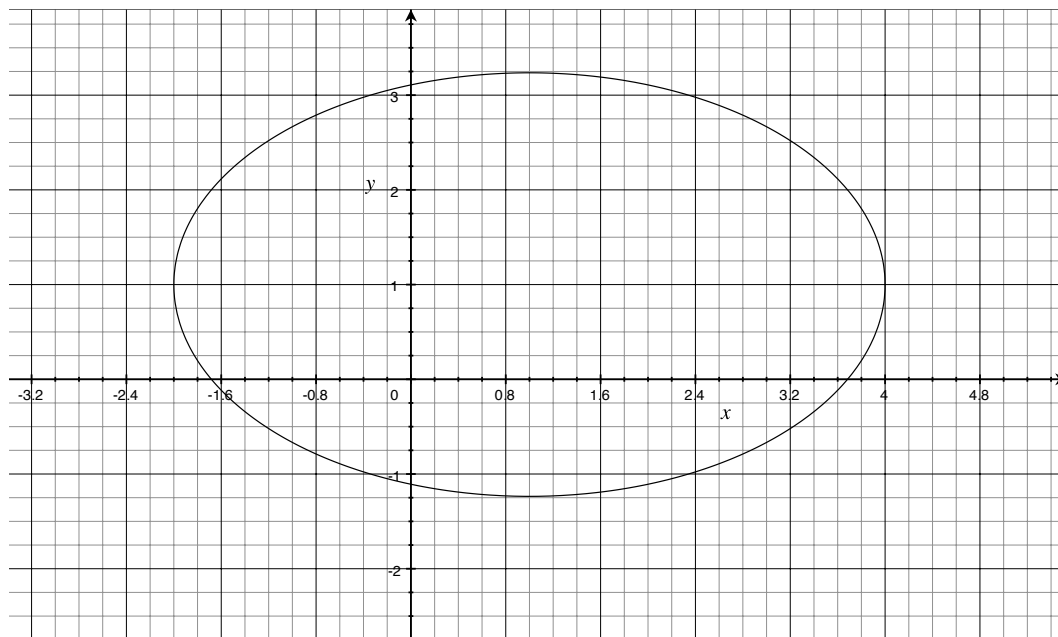
Since the sum of the distances to the foci must be 6, we know that  $2a = 6$  (draw a picture to

convince yourself, or do some algebra as in class Wednesday.5/6).

So,  $a = 3$  and the vertices are  $(4, 1)$  and  $(-2, 1)$ .

Thus,  $y_0 = 1$ ,  $x_0 = 1$ ,  $c = 2$ ,  $a = 3$ , and  $b = \sqrt{5}$ . The equation is

$$\frac{(x - 1)^2}{9} + \frac{(y - 1)^2}{5} = 1$$



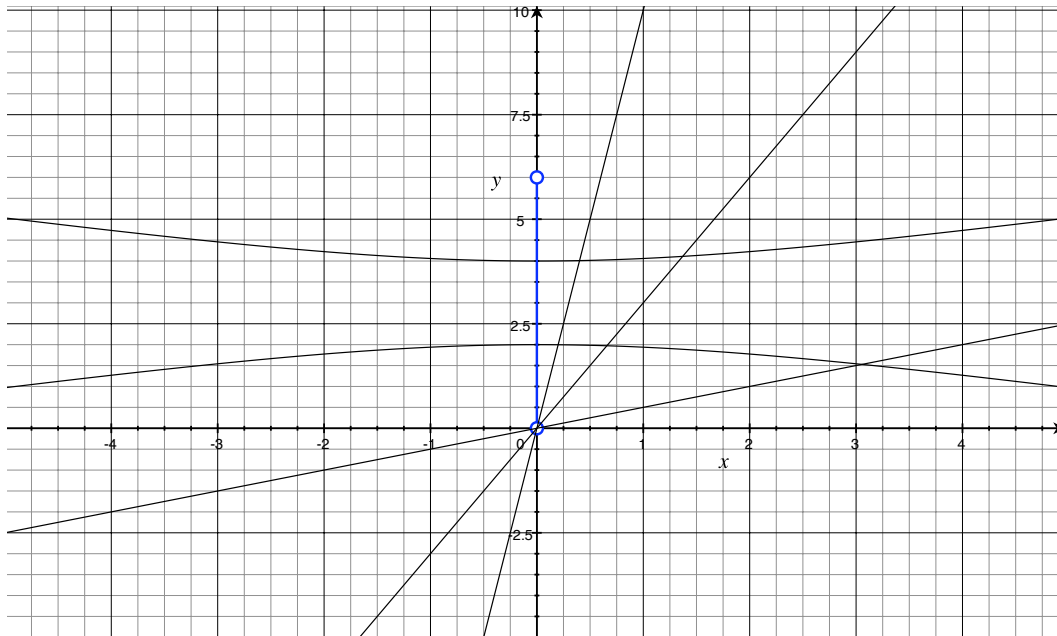
#38. If a hyperbolic mirror is in the shape of the bottom half of

$$(y - 3)^2 - \frac{x^2}{8} = 1$$

to what point will the rays of light following the path  $y = cx$  ( $y > 0$ ) reflect?

ANSWER

We may figure out from the equation that the center is  $(0, 3)$ , so  $x_0 = 1$  and  $y_0 = 3$ , and we may see that  $a = 1$ ,  $b = \sqrt{8}$ , which makes  $c = 3$ . So, the foci of this hyperbola are  $(0, 0)$  and  $(0, 6)$ . Light rays following the path  $y = cx$  would go through the focus  $(0, 0)$ , so they would be reflected toward the focus at  $(0, 6)$ .



In the picture, imagine that the top half of the hyperbola isn't there, and that the upper side of the bottom half of the hyperbola is a mirror. The lines which go thru the focus  $(0,0)$  represent the rays of light, and imagine they are coming from above (for  $y > 0$ ). When they hit the mirror, they bounce off and instead of actually going thru  $(0,0)$ , they get reflected towards  $(0,6)$ . (The reflection isn't shown in the picture, but hopefully you can imagine it or draw it in).

#42. If the ceiling of a room has the shape

$$\frac{x^2}{900} + \frac{y^2}{100} = 1$$

where should you place two desks so that you can sit at one desk and hear everything said at the other desk?

ANSWER

We should place the desks at the foci of the ellipse. We can see that the center of the ellipse will be  $(0,0)$ , so  $x_0 = 0$  and  $y_0 = 0$ . Also,  $a^2 = 900$  and  $b^2 = 100$ , so  $c = \sqrt{900 - 100} = 20\sqrt{2}$ . Then, the foci are at  $(\pm 20\sqrt{2}, 0)$ .