MATH 231 U1, Spring 2009
Answers to HW 4, Worksheet 1, 3, 4 and Section 6.3 problems 13, 14
Due Friday January 30th, 2009

Worksheet #1 $\int \arctan x \frac{e^{\arctan x}}{1+x^2}\, dx$

Start with a substitution $a = \arctan x$, $da = \frac{1}{1+x^2}\, dx$

$$\int \arctan x \frac{e^{\arctan x}}{1+x^2}\, dx = \int ae^a\, da$$

Now, use IBP (Integration By Parts) with $u = a$ and $dv = e^a\, da$, which gives $du = da$ and $v = e^a$.

$$\int ae^a\, da = ae^a - \int e^a da = ae^a - e^a + C$$

Putting our answer back in terms of x we get

$$\int \arctan x \frac{e^{\arctan x}}{1+x^2}\, dx = (\arctan x)e^{\arctan x} - e^{\arctan x} + C$$

#3 $\int \arctan x\, dx$

Use IBP with $u = \arctan x$ and $dv = dx$, then $du = \frac{1}{1+x^2}\, dx$ and $v = x$, so we get

$$\begin{aligned}\int \arctan x\, dx &= x \arctan x - \int \frac{x}{1+x^2}\, dx \\ &= x \arctan x - \frac{1}{2} \ln|1+x^2| + C\end{aligned}$$

The last step can be done via a substitution with $u = 1 + x^2$.

#4 $\int \cos x^{1/3}\, dx$

Use substitution with $u = x^{1/3}$, then $du = \frac{1}{3}x^{-2/3}\, dx$, notice we can solve for $dx = 3u^2\, du$ easily in terms of u. This is one clue that our method is on the right track. We get:

$$\int \cos x^{1/3}\, dx = 3 \int u^2 \cos u\, du$$

Now we can use IBP twice to get our answer. Let $u' = u^2$ and $dv' = \cos u\, du$, making $du' = 2u\, du$ and $v' = \sin u$.

$$\begin{aligned}
3\int u^2\cos u\ du &= 3(u^2\sin u - 2\int u\sin u\ du)\\
&= 3(u^2\sin u - 2(-u\cos u + \int \cos u\ du))\\
&= 3(u^2\sin u + 2u\cos u - 2\sin u) + C\\
&= 3x^{2/3}\sin x^{1/3} + 6x^{1/3}\cos x^{1/3} - 6\sin x^{1/3} + C
\end{aligned}$$

For the second use of IBP, we made $u' = u$ and $dv' = \sin u$, so $du' = du$ and $v = -\cos u$.

6.3 #13: $\int \sin^2 x\cos^2 x\ dx$

Use the half angle formulas

$$\begin{aligned}
\int \sin^2 x\cos^2 x\ dx &= \int \frac{1}{2}(1-\cos 2x)\frac{1}{2}(1+\cos 2x)\ dx\\
&= \int \frac{1}{4}(1-\cos^2(2x))\ dx\\
&= \frac{1}{4}\int (1 - \frac{1}{2}(1+\cos 4x))\ dx\\
&= \frac{1}{4}(x - \frac{1}{2}x - \frac{\sin 4x}{8}) + C
\end{aligned}$$

#14: $\int (\sin^2 x + \cos^2 x)\ dx$

$$\int (\sin^2 x + \cos^2 x)\ dx = \int 1\ dx = x + C$$