MATH 231 U1, Spring 2009
Exam 1: 6.1-6.6, 7.1 B
Wednesday February 18th, 2009

NAME: ANSWER KEY
(Please Print)

Seat Assignment:

Back of Classroom

Windows	1	2	3	4	5	6	7	Door
	1	2	3	4	5	6	7	
	1	2	3	4	5	6	7	
	1	2	3	4	5	6	7	
	1	2	3	4	5	6	7	

Front of Classroom

DIRECTIONS:

- Do not open Exam until instructed to do so.
- Sit in the seat indicated above.
- Do each of the problems and show <u>**all**</u> work as the work leading to the solution is far more important than the actual answer.
- **NO WORK MEANS NO POINTS!**
- Calculators **ARE NOT ALLOWED** on this Exam.
- Box or circle and **LABEL** your final solution.
- You will have 50 minutes to complete this Exam.

1. (a) (4 points) State the integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

(b) (4 points) What is wrong with the choice of u and dv in the following attempted use of Integration by Parts? Explain. (You DO NOT have to calculate the integral here.)

To do $\int \tan^3 x \sec^3 x\, dx$, *let* $u = \tan x$ *and* $dv = \sec^2 x\, dx$.

For this choice of u and dv

$$\int u\,dv = \int \tan x \sec^2 x\, dx \neq \int \tan^3 x \sec^3 x\, dx.$$

We have a $\tan^2 x$ and a $\sec x$ in the integrand which are <u>not</u> in $u\,dv$.

1(c) (10 points) Calculate $\int_e^{e^2} \frac{\ln x}{x^2}\, dx$

let $u = \ln x$ and $dv = x^{-2} dx$

then $du = \frac{1}{x} dx$ $\quad v = -x^{-1}$

$$\int_e^{e^2} \frac{\ln x}{x^2}\, dx = \left.\frac{-\ln x}{x}\right|_e^{e^2} + \int_e^{e^2} \frac{1}{x^2}\, dx$$

$$= \left[\frac{-\ln e^2}{e^2} + \frac{\ln e}{e}\right] + \left[-x^{-1}\Big|_e^{e^2}\right]$$

$$= \left[\frac{-2}{e^2} + \frac{1}{e}\right] + \left[-\frac{1}{e^2} + \frac{1}{e}\right]$$

OK to stop here, you don't have to simplify in this problem.

$$= \boxed{\frac{-3}{e^2} + \frac{2}{e}}$$

2. (10 points) Write down the partial fractions decomposition for

$$\frac{4x^2 - 2x + 1}{x^3(x^4 - 1)}$$

with the unknowns A, B, C etc. STOP. DO NOT solve for A, B, C etc.

First: You must factor the denominator into <u>linear</u> and <u>irreducible</u> <u>quadratic</u> factors

$$x^3(x^4-1) = x^3(x^2+1)(x^2-1) = x^3(x^2+1)(x+1)(x-1)$$

x is a linear factor $\quad$ irred. $\quad$ <u>not</u> irred. $\quad$ linear $\quad$ linear

$$\frac{4x^2-2x+1}{x^3(x^4-1)} = \boxed{\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{F}{x+1} + \frac{G}{x-1}}$$

3. (14 points) Find $\int \frac{1}{(\sqrt{25-x^2})^3}\,dx$. Show all your work.

let $x = 5\sin\theta$ for θ in $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\left(\theta \neq \frac{\pi}{2} \text{ or } -\frac{\pi}{2}\right.$ b/c we'd be dividing by $0\left.\right)$

then $dx = 5\cos\theta\, d\theta$

So

$$\int \frac{1}{(\sqrt{25-x^2})^3}dx = \int \frac{5\cos\theta}{(\sqrt{25-25\sin^2\theta})^3}d\theta$$

$$= \int \frac{5\cos\theta}{(5\sqrt{1-\sin^2\theta})^3}d\theta$$

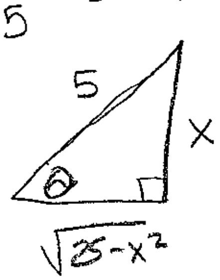
$$= \frac{5}{5^3}\int \frac{\cos\theta}{(\sqrt{1-\sin^2\theta})^3}d\theta$$

$$= \frac{1}{5^2}\int \frac{\cos\theta}{(\cos\theta)^3}d\theta$$

$$= \frac{1}{25}\int \frac{1}{\cos^2\theta}d\theta$$

$\frac{x}{5} = \sin\theta$

$$= \frac{1}{25}\int \sec^2\theta\, d\theta$$



$$= \frac{1}{25}\tan\theta + C$$

so $\tan\theta = \frac{x}{\sqrt{25-x^2}}$

$$\boxed{= \frac{1}{25}\frac{x}{\sqrt{25-x^2}} + C}$$

4. (10 points) (a) Say why the following integral is improper: $\int_1^2 \frac{1}{(x-1)^{2/3}}\,dx$

because $\frac{1}{(x-1)^{2/3}}$ has a vertical asymptote at $x=1$, so $\frac{1}{(x-1)^{2/3}}$ is <u>not</u> continuous on $[1,2]$, and $\lim_{x\to 1^+}\left|\frac{1}{(x-1)^{2/3}}\right| = \infty$.

(b) Evaluate $\int_1^2 \frac{1}{(x-1)^{2/3}}\,dx$. Show all your work.

$$= \lim_{R\to 1^+} \int_R^2 \frac{1}{(x-1)^{2/3}}\,dx = \lim_{R\to 1^+} \int_R^2 (x-1)^{-2/3}\,dx$$

$$= \lim_{R\to 1^+} 3(x-1)^{1/3}\Big|_R^2 = \lim_{R\to 1^+}\left(3(1)^{1/3} - 3(R-1)^{1/3}\right)$$

$$= 3 - \lim_{R\to 1^+} 3(R-1)^{1/3} = 3 - 0 = \boxed{3}$$

5. (14 points) Say whether the following improper integrals converge or diverge, and explain why you know.
(You DON'T need to calculate their values.)

(i) $\int_1^\infty e^{-x^5}\,dx$

Compare to $\int_1^\infty e^{-x}\,dx$

$e^x \le e^{x^5}$ so $\frac{1}{e^x} = e^{-x} \geq e^{-x^5} = \frac{1}{e^{x^5}}$ and $\boxed{0 \le e^{-x^5} \le e^{-x}}$

Showing they are both positive is important, since we can only apply the comparison test to positive functions!

We know $\int_1^\infty e^{-x}\,dx$ converges

therefore $\int_1^\infty e^{-x^5}\,dx$ converges

by the comparison test.

(ii) $\int_0^{17} \frac{1+\sin^2 x}{x^2}\,dx$

Compare to $\int_0^{17} \frac{1}{x^2}\,dx$.

$\int_0^{17} \frac{1}{x^2}\,dx$ diverges by the p-test for integrals like $\int_0^a \frac{1}{x^p}\,dx$

and

$$\boxed{0 \le} \frac{1}{x^2} \le \frac{1+\sin^2 x}{x^2}$$

because $1 \le 1+\sin^2 x$, since $\sin^2 x \geq 0$.

Therefore $\int_0^{17} \frac{1+\sin^2 x}{x^2}\,dx$ diverges too,

by the comparison test.

6. (18 points) (a) Find the specific solution to this differential equation:

$$y' = 3y \qquad y(0) = -2$$

$y = Ae^{3t}$

$y(0) = Ae^{0} = A = -2$

$\boxed{y(t) = -2e^{3t}}$

(b) Once you mix Love Potion # 9 with another liquid, it decays at an exponential rate, with a half-life of 3 minutes. Say you give the object of your affection a cup of soda dosed with 12mg of Love Potion #9.

Give a function which tells you the milligrams of Love Potion left in the drink after t minutes.

$P(t) = 12e^{kt}$

$6 = P(3) = 12e^{k \cdot 3}$

$\frac{1}{2} = e^{k3}$

$\ln\frac{1}{2} = k3$

$\frac{\ln\frac{1}{2}}{3} = k \longrightarrow$ So $P(t) = 12e^{\left(\frac{\ln\frac{1}{2}}{3}\right)t}$

7. Calculate the following 2 integrals. Use any method you like.

(a) (8 points) $\int \cos^2 x \, dx$

$$= \int \frac{1}{2}(1+\cos 2x)\,dx = \frac{1}{2}\int (1+\cos 2x)\,dx$$

by half-angle formula

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$$

7.(b) (8 points) $\int \frac{4}{x^3+x}\,dx$ Use partial fractions!

$$= \int \left[\frac{4}{x} - \frac{4x}{x^2+1}\right] dx$$

$$\frac{4}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

linear irred. quadratic

$$= 4\ln|x| - 2\ln|x^2+1| + C$$

to do $\int \frac{4x}{x^2+1}\,dx$

you can use substitution

$u = x^2+1$

$du = 2x$

$$\int \frac{4x}{x^2+1}\,dx = 2\int \frac{1}{u}\,du$$

$$= 2\ln|u|$$

$$= 2\ln|x^2+1|$$

$$4 = Ax^2 + A + Bx^2 + Cx$$

$\therefore$ $A = 4$

$B = -A = -4$

$C = 0$