

# Math 231, Section U1

## Exam 3 Checklist

### Sections 8.6-8.8, 9.1-9.3

#### Introduction

This is an outline of material I expect you to know for Exam 3. It is not guaranteed to be comprehensive, and details are omitted, but it should give you a good idea of what to concentrate on. Problems from the quizzes, examples in class and the problems from the homework should also give you a good idea of what material will be covered on the test. I recommend looking at the "writing exercises" in addition to the problems, as they will help you understand the concepts. Also check out the relevant chapter review exercises. To prepare for the test, you should get lots of practice and be sure you understand why you are doing what you are doing! Get help ASAP if you need it.

#### Background

Math is cumulative. I expect you to remember and understand all the relevant background material, from this class, from Calc I and before. Specifically, you may need some of the material in 8.1-8.5 in order to do problems from 8.6-8.8 including, the Ratio test, geometric series, alternating series and the alternating series error estimate. You also will need to dust off your differentiation and integration skills!

#### Section 8.6: Power Series

- Know the definition of a power series and how to recognize whether a series is a power series or not, know that a power series is a function of  $x$ .
- Know what the terms *center*, *coefficient*, *radius of convergence*, and *interval of convergence* mean
- Know how to recognize a geometric series, find its radius and interval of convergence and find what function it converges to, and be able to do this backwards too (that is, given an appropriate function and a specified center, find the corresponding geometric series)
- Know how to use the Ratio test to find the radius of convergence, and how to find the interval of convergence for a power series (don't forget to test the endpoints!)
- Know and understand Theorem 6.1. You need not memorize it word for word but you need to know it and be able to answer questions about what it means.
- Know how to differentiate power series and integrate power series, and know how to use this to find power series representations for appropriate functions. Know what happens to the radius and interval of convergence!

#### Section 8.7: Taylor Series

- Know what a Taylor series is, and what a Maclaurin series is, and that both of these are examples of power series representations of functions, know how to find their intervals of convergence
- Know that for a function  $f(x)$  and a center  $c$  there is at most ONE power series representation for that function at that center.
- Know what a Taylor polynomial of degree  $n$  (denoted  $P_n(x)$ ) is, what they are used for and how to use them!
- Know and understand the formula for the coefficients of Taylor series and Taylor polynomials, and know how to use it.
- Know Taylor's Theorem (be able to state it!) and how to use it to estimate the error when using
- Know Theorem 7.2, and why it is relevant
- If you want to use Taylor series to approximate something, know how to pick an appropriate center for the series
- Memorize the Taylor series for:
  - $\sin x$  centered at 0
  - $\cos x$  centered at 0
  - $e^x$  centered at 0
  - $\ln x$  centered at 1
  - $\tan^{-1}(x)$  centered at 0

AND know their intervals of convergence!

- Know how to find “new” Taylor series from old ones via addition, subtraction, multiplication, ”plugging in”, differentiation, integration, and know what happens to the interval of convergence!

### Section 8.8: Applications of Taylor Series

- Know how to use Taylor series to approximate numbers, and how to estimate the error (using Taylor's Theorem, or using the AST error test where appropriate.)
- Know how to use Taylor series to approximate the value of an integral, and how to do this in the context of a word problem (e.g. the Tribbles problem)
- Know how to use Taylor series to find the value of a limit
- Know how to choose an appropriate center for your Taylor series when doing all the above problems.

## Section 9.1: Parametric Equations

- Know what parametric equations are, and how to sketch the curves they define by hand (for simple cases)
- Know what a plane curve is, know what is meant by the *orientation* of a curve given by parametric equations
- Know how to eliminate the parameter
- Know how to find the parametric equations for circles, ellipses, line segments, parabolas and other easy examples
- Know how to match a set of parametric equations with its graph (like problems 25-30)
- Know that there are infinitely many different possible parameterizations of any given curve
- Know how to figure out if two curves defined by parametric equations intersect
- Know the difference between just the curves intersecting, and the situation in the Missile example (1.9), where not only do the curves need to intersect, but the intersection point must correspond to the same moment in time.

## Section 9.2: Calculus with Parametric Equations

- Know how to find  $\frac{dy}{dx}$  for a curve given by parametric equations, recall that  $\frac{dy}{dx}|_{(x,y)}$  is the slope of the tangent line to the curve at the point  $(x, y)$ , this is also written  $\frac{dy}{dx}|_{t=a}$ , which gives the slope of the tangent line to the curve at the point  $(x(a), y(a))$ .
- Know the difference between velocity and speed, and know how to find the speed and direction of an object whose position is given by parametric equations
- Know how to find the area enclosed by a curve defined parametrically

## Section 9.3: Arc Length and Surface Area in Parametric Equations

- Know what arc length means
- Know how to find arc length for a curve defined parametrically
- Understand the brachistochrone problem (Example 3.3)
- Know what a cycloid is
- Know how to find surface area for a solid of revolution of a curve defined parametrically
- Be able to model simple physical situations with parametric equations (for example, giving the position of some kind of object in space.)