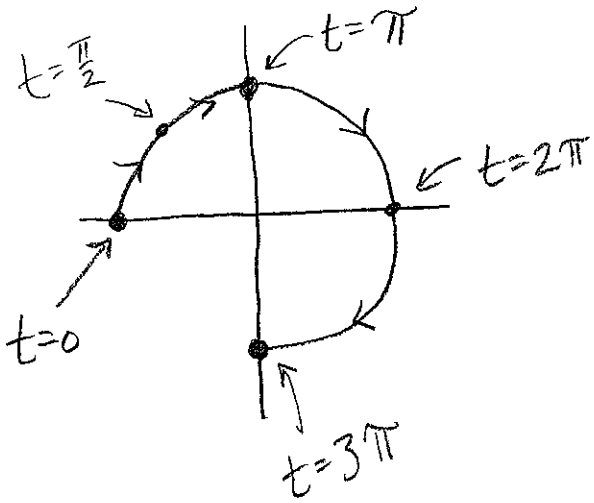
MATH 231 U1, Spring 2009
Quiz 10, Version 1
Friday April 17th, 2009

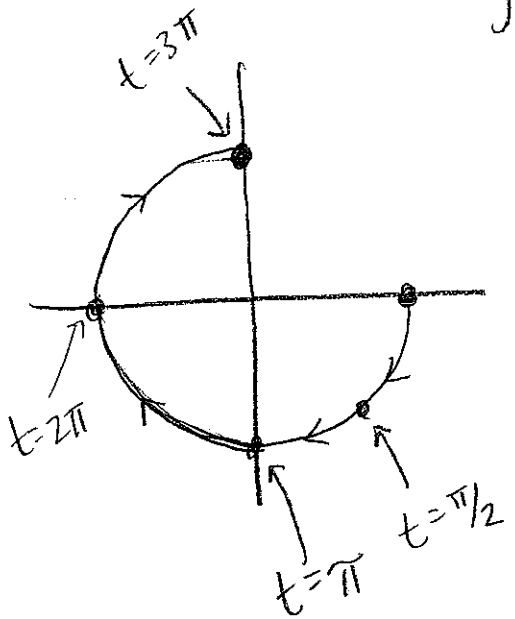
1. Sketch the curve given by the parametric equations $x(t) = -\cos(\frac{t}{2})$ and $y(t) = \sin(\frac{t}{2})$, for $t \in [0, 3\pi]$. On your graph, label the point which corresponds to $t = 0$, the point which corresponds to $t = \pi$ and be sure to indicate the *orientation* of the curve.



All the points will be on the circle of radius 1 centered at (0, 0).

t	(x, y)
0	$(-1, 0)$
$\frac{\pi}{2}$	$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
π	$(0, 1)$
2π	$(1, 0)$
3π	$(0, -1)$

Version 2 $\quad x(t) = \cos(\frac{t}{2})$ $\quad y(t) = -\sin(\frac{t}{2})$ $\quad t$ in $[0, 3\pi]$



t	(x, y)
0	$(1, 0)$
$\frac{\pi}{2}$	$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
π	$(0, -1)$
2π	$(-1, 0)$
3π	$(0, 1)$

2. (a) Ant # 1 is crawling across your paper (at a constant speed) along the line segment from $(1,1)$ to $(5,5)$ for time t in $[0,4]$. What are the parametric equations describing the position of the Ant #1? (That is, find linear parametric equations for the line segment starting at $(1,1)$ when $t=0$ and ending at $(5,5)$ when $t=4$.)

$x = a+bt$ when $t=0$, $x=1$, $y=1$

$y = c+dt$ when $t=4$ $x=5$, $y=5$

Plug in 0 to see: So $a=1$ $c=1$

$x = 1+bt$

$y = 1+dt$ then plug in 4 $x = 1+4b$

When $t=4$, $x=y=5$ $y = 1+4d$

So $b=1$ and $d=1$

$\begin{cases} x = 1+t & t \text{ in} \\ y = 1+t & [0,4] \end{cases}$

(b) After 1 second, Ant #2 starts crawling from $(2,4)$ towards $(5,1)$ along a line segment. The position of Ant #2 is given by:

$$x(t) = (t-1)+2, \quad y(t) = 4-(t-1) \quad \text{for } t \in [1,4]$$

Do these ants collide? If so, when and where do they collide?

Set the x-equations equal

$1+t = (t-1)+2 = 1+t \Rightarrow t=t$

this means the x-coordinates are equal for all t in $[0,4]$

Set the y-eq^{n}s equal

$1+t = 4-(t-1) = 5-t \Rightarrow 2t=4 \Rightarrow t=2$

So when $t=2$ both ants are at the point $(3,3)$ and there is a collision!