MATH 231 U1, Spring 2009
Quiz 2, Version 1
Friday January 30th, 2009

1. (a) State the integration by parts formula

$$\int u\ dv = uv - \int v\ du$$

(or for definite integrals:)

$$\int_a^b u\ dv = uv|_a^b - \int_a^b v\ du$$

(b) (Version 1) Find $\displaystyle\int xe^x\ dx$

Use IBP with $u = x$ and $dv = e^x\ dx$. Then $du = dx$ and $v = e^x$ and we get

$$\int xe^x\ dx = xe^x - \int e^x\ dx = xe^x - e^x + C$$

(b) (Version 2) Find $\displaystyle\int x \ln x\ dx$

Use IBP with $u = \ln x$ and $dv = x\ dx$. Then $du = \frac{1}{x}\ dx$ and $v = \frac{1}{2}x^2$ and we get

$$\begin{aligned}\int x \ln x\ dx &= \frac{x^2}{2}\ln x - \int \frac{1}{2}x^2\frac{1}{x}\ dx \\ &= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\ dx \\ &= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C\end{aligned}$$

2. Find $\displaystyle\int \sin x \sin 2x\ dx$

One way to do this problem is to use the trig identity $\sin 2x = 2 \sin x \cos x$. Then, do substitution with $u = \sin x$, so $du = \cos x\ dx$.

$$\begin{aligned}\int \sin x \sin 2x\ dx &= \int 2\sin^2 x \cos x\ dx \\ &= 2\int u^2\ du \\ &= \frac{2}{3}u^3 + C \\ &= \frac{2}{3}\sin^3 x + C\end{aligned}$$

The other way to do this problem is to do IBP twice and then solve.
We will pick $u = \sin 2x$ and $dv = \sin x\ dx$. So then $du = 2\cos 2x\ dx$ and $v = -\cos x$.
Then, to get to the 2nd line from the 1st line, we do another integration by parts, choosing $u = \cos 2x$ and $dv = \cos x\ dx$, making $du = -2\sin 2x\ dx$ and $v = \sin x$

$$\begin{aligned}
\int \sin x \sin 2x\ dx &= -\sin 2x \cos x + 2\int \cos x \cos 2x\ dx \\
&= -\sin 2x \cos x + 2(\cos 2x \sin x + 2\int \sin x \sin 2x\ dx) \\
\int \sin x \sin 2x\ dx &= -\sin 2x \cos x + 2\cos 2x \sin x + 4\int \sin x \sin 2x\ dx \\
\text{so we get } -3\int \sin x \sin 2x\ dx &= -\sin 2x \cos x + 2\cos 2x \sin x \\
\int \sin x \sin 2x\ dx &= \frac{1}{3}\sin 2x \cos x - \frac{2}{3}\cos 2x \sin x + C
\end{aligned}$$

The two answers from these two methods are actually equal, you can use trig identities to prove it.