MATH 231 U1, Spring 2009
Quiz 3, Version 1 KEY
Friday February 6th, 2009
For full credit, show your work and justify your answers.

(5pts) 1. Find $\int x^3\sqrt{x^2+9}\,dx$

let $x = 3\tan\theta$ for $\rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

then $dx = 3\sec^2\theta\,d\theta$

$$\int x^3\sqrt{x^2+9}\,dx = \int 3^3\tan^3\theta\sqrt{9\tan^2\theta+9}\;3\sec^2\theta\,d\theta$$

$$= 3^5\int \tan^3\theta\sqrt{\sec^2\theta}\,\sec^2\theta\,d\theta = 3^5\int \tan^3\theta\sec^3\theta\,d\theta$$

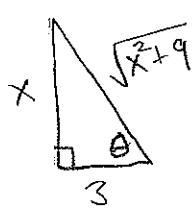
$$= 3^5\int(\sec^2\theta-1)\sec^2\theta\tan\theta\sec\theta\,d\theta$$

let $u = \sec\theta$
$du = \tan\theta\sec\theta\,d\theta$

$$= 3^5\int(u^4-u^2)\,du = 3^5\left[\frac{u^5}{5}-\frac{u^3}{3}\right]+C$$

$$= 3^5\left[\frac{\sec^5\theta}{5}-\frac{\sec^3\theta}{3}\right]+C$$

$\frac{x}{3} = \tan\theta$ so



$\sec\theta = \frac{\sqrt{x^2+9}}{3}$

$$= 3^5\left[\frac{(x^2+9)^{5/2}}{3^5\cdot 5}-\frac{(x^2+9)^{3/2}}{3^4}\right]+C$$

(2pts) 2. a) Write down the partial fraction decomposition for

$$\frac{3x+7}{x^3(x^2+4)}$$

with unknowns A, B, C etc. (DO NOT set up the system of equations, and DO NOT solve for A, B, C, etc.)

$$\frac{3x+7}{x^3(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4}$$

(3pts) b) Calculate the following integral.

$$\int \frac{3}{x^2+x}\,dx$$

$$\frac{3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$A(x+1) + Bx = 3$$

$$\Rightarrow A = 3 \quad B = -3$$

$$= \int \frac{3}{x}\,dx + \int \frac{-3}{x+1}\,dx$$

$$= 3\ln|x| + (-3)\ln|x+1| + C$$