MATH 231 U1, Spring 2009
Quiz 4, Version 1
Friday February 13th, 2009

ANSWERS

1. (a) Is the following integral improper? Why or why not?

$$\int_2^7 \frac{1}{\sqrt{x-2}}\,dx$$

YES. The integrand $\frac{1}{\sqrt{x-2}}$ is not continuous at 2.
(It has a vertical asymptote at 2)

(b) Evaluate $\int_2^7 \frac{1}{\sqrt{x-2}}\,dx$. Show all your work.

use substitution if you need to

$$\int_2^7 \frac{1}{\sqrt{x-2}}\,dx = \lim_{R\to 2^+} \int_R^7 \frac{1}{\sqrt{x-2}}\,dx = \lim_{R\to 2^+} \left[2(x-2)^{1/2}\right]_R^7$$

$$= \lim_{R\to 2^+} \left[2(5)^{1/2} - 2(R-2)^{1/2}\right]$$

$$= 2\sqrt{5} - 0 = \boxed{2\sqrt{5}}$$

must write this step!
You Need to use the limit.

2. Apply a COMPARISON TEST to answer the following multiple choice questions. For each one, choose which of the 3 responses is best and SHOW SOME WORK which justifies your answer! The point of this question is to demonstrate your understanding of the comparison test.

(a) We know that $\int_0^1 \frac{1}{x^2}\,dx$ diverges. Using the comparison test, this implies that $\int_0^1 \frac{1}{x+x^2}\,dx$

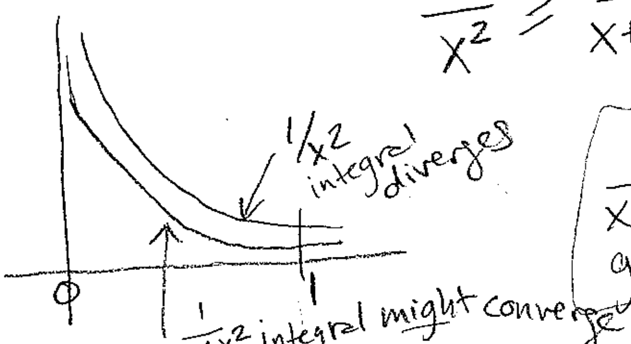
(i) converges
(ii) diverges
$\boxed{\text{(iii) comparing } \int_0^1 \frac{1}{x^2}\,dx \text{ and } \int_0^1 \frac{1}{x+x^2}\,dx \text{ is inconclusive}}$

For x in $[0,1]$, $x^2 \le x + x^2$

so $\dfrac{1}{x^2} \ge \dfrac{1}{x+x^2}$.

But, $\int_0^1 \frac{1}{x^2}dx$ diverges

So showing our function $\frac{1}{x+x^2}$ is less than $\frac{1}{x^2}$ does not give us a conclusion about $\int_0^1 \frac{1}{x^2+x}dx$



(b) We know that $\int_1^\infty \frac{1}{e^x}\,dx$ converges. Using the comparison test, this implies that $\int_1^\infty \frac{\sin^2 x}{x+e^x}\,dx$

$\boxed{\text{(i) converges}}$
(ii) diverges
(iii) the comparison is inconclusive

For x in $[1,\infty)$, $x+e^x \ge e^x$

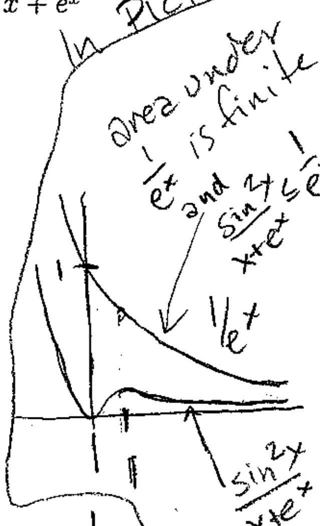
and $0 \le \sin^2 x \le 1$.

So $\dfrac{\sin^2 x}{x+e^x} \le \dfrac{1}{x+e^x} \le \dfrac{1}{e^x}$

$\int_1^\infty \frac{1}{e^x}dx$ converges and $\frac{\sin^2 x}{x+e^x} \le \frac{1}{e^x}$,

therefore $\int_1^\infty \frac{\sin^2 x}{x+e^x}dx$ converges too

In PICTURES:



so area under $\frac{\sin^2 x}{x+e^x}$ is finite too