MATH 231 U1, Spring 2009
Quiz 5, Version 1 ANSWERS
Friday February 27th, 2009

1. (a) Does the sequence $\left\{\frac{7n^2+2n}{3n^2+4}\right\}_{n=1}^{\infty}$ converge? If it converges, find its limit as $n \to \infty$. Be sure to show work.

ANSWER

$$\lim_{n\to\infty} \frac{7n^2+2n}{3n^2+4} = \lim_{n\to\infty} \frac{7+\frac{2}{n}}{3+\frac{4}{n^2}} = \frac{7}{3}.$$

So, $\lim_{n\to\infty} \frac{7n^2+2n}{3n^2+4} = \frac{7}{3}$.

(Alternatively, you could use L'Hopital's rule to find the limit $\lim_{x\to\infty} \frac{7x^2+2x}{3x^2+4}$ plus the theorem that says if $\lim_{x\to\infty} f(x) = L$ then $\lim_{n\to\infty} f(n) = L$ also.)

(b) Does the sequence $\left\{(-1)^n \frac{7n^2+2n}{3n^2+4}\right\}_{n=1}^{\infty}$ converge or diverge? How do you know? Justify your answer.

ANSWER

This sequence $a_n = (-1)^n \frac{7n^2+2n}{3n^2+4}$ DIVERGES.

We showed above that the sequence of absolute values $|a_n| = \frac{7n^2+2n}{3n^2+4}$ converges to $\frac{7}{3}$.

Therefore, the odd terms of this sequence $a_n = (-1)^n \frac{7n^2+2n}{3n^2+4}$ will have limit $-\frac{7}{3}$ and the even terms will have limit $\frac{7}{3}$, and there is no way the sequence can converge.

This is a fact that we stated in class: if $\lim_{n\to\infty} |a_n| \neq 0$, then the sequence $(-1)^n |a_n|$ diverges.