MATH 231 U1, Spring 2009
Quiz 6, Version 1
Friday March 6th, 2009

1. Do the following series converge or diverge? Justify your answers! Tell me what tests or rules you are using.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{\cancel{k^2+1}}{\cancel{(k^2+1)}k^{1.04}}$ $= \displaystyle\sum_{K=1}^{\infty} \frac{1}{K^{1.04}}$ $1.04 > 1$ so this p-series Converges by the "p-test."

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3+5k^{3/2}-k+4}{k^3+5\sqrt{k}+2}$

$$\lim_{K\to\infty} \frac{K^3+5K^{3/2}-K+4}{K^3+5\sqrt{K}+2} \frac{\left(\frac{1}{K^3}\right)}{\left(\frac{1}{K^3}\right)}$$

$$= \lim_{K\to\infty} \frac{1+\frac{5}{K^{3/2}}-\frac{1}{K^2}+\frac{4}{K^3}}{1+\frac{5}{K^{5/2}}+\frac{2}{K^3}} = 1 \neq 0$$

So the series diverges by the K^{th} Term Test.

(c) $\displaystyle\sum_{k=3}^{\infty} \frac{\ln k}{k}$

$\ln K \geq 1$ for $K \geq 3$ so $0 \geq \frac{\ln K}{K} \geq \frac{1}{K}$ for $K \geq 3$

$\boxed{\text{Don't forget this part!}}$

we know $\displaystyle\sum_{K=3}^{\infty} \frac{1}{K}$ diverges

b/c the harmonic series diverges

Therefore by the comparison Test $\displaystyle\sum_{K=3}^{\infty} \frac{\ln K}{K}$ diverges too.

2. Find the sum of $\sum_{k=138}^{\infty} (-1)^k \frac{5}{17^k}$. (DO NOT SIMPLIFY THE ANSWER)

Write out some terms to see this is $\boxed{\text{geometric}}$!

$$\sum_{k=138}^{\infty} (-1)^k \frac{5}{17^k} = \frac{5}{17^{138}} - \frac{5}{17^{139}} + \frac{5}{17^{140}} - \frac{5}{17^{141}} + \cdots$$

the ratio is $r = \frac{a_{k+1}}{a_k} = \frac{(-1)^{k+1} \frac{5}{17^{k+1}}}{(-1)^k \frac{5}{17^k}} = -\frac{1}{17}$

the first term a is $\frac{5}{17^{138}}$.

$|r| = \frac{1}{17} < 1$ so this geometric series converges.

So $$\sum_{k=138}^{\infty} (-1)^k \frac{5}{17^k} = \frac{a}{1-r} = \boxed{\frac{\frac{5}{17^{138}}}{1+\frac{1}{17}}}$$

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Quiz 6, Version 2
Friday March 6th, 2009

1. Do the following series converge or diverge? Justify your answers! Tell me what tests or rules you are using.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{\cancel{k^2+2}}{\cancel{(k^2+2)}k^{0.98}} = \sum_{K=1}^{\infty} \frac{1}{K^{0.98}}$

$0.98 \le 1$ so this p-series Series diverges by the p-test

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3+5k^{3/2}+4}{k^5+7\sqrt{k}+2}$ $= a_K$

let $b_K = \frac{1}{K^2}$, we know $a_K, b_K > 0$.

$$\lim_{K\to\infty} \frac{a_K}{b_K} = \lim_{K\to\infty} \frac{K^3+5K^{3/2}+4}{K^5+7\sqrt{K}+2} \cdot \frac{K^2}{1} = \lim_{K\to\infty} \frac{K^5+5K^{7/2}+4K^2}{K^5+7\sqrt{K}+2} \frac{\left(\frac{1}{K^5}\right)}{\left(\frac{1}{K^5}\right)}$$

$$= \lim_{K\to\infty} \frac{1+\frac{5}{K^{3/2}}+\frac{4}{K^3}}{1+\frac{7}{K^{9/2}}+\frac{2}{K^5}} = 1 > 0$$

So, since $\sum_{K=1}^{\infty} \frac{1}{K^2}$ converges by the p-test, this series converges by the Limit Comparison Test.

(c) $\displaystyle\sum_{k=3}^{\infty} \frac{e^k}{k}$

$$\lim_{K\to\infty} \frac{e^K}{K} = \lim_{x\to\infty} \frac{e^x}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{1} = \infty$$

So this <u>series diverges by the K^{th} term test</u>

2. Find the sum of $\sum_{k=72}^{\infty}(-1)^k \frac{3}{11^k}$. (DO NOT SIMPLIFY THE ANSWER)

Write out some terms to see that this is geometric!

$$\sum_{k=72}^{\infty} (-1)\frac{3}{11^k} = \frac{3}{11^{72}} - \frac{3}{11^{73}} + \frac{3}{11^{74}} - \frac{3}{11^{75}} + \cdots$$

$$r = \frac{a_{k+1}}{a_k} = \frac{(-1)^{k+1}\frac{3}{11^{k+1}}}{(-1)^k \frac{3}{11^k}} = \frac{-1}{11}$$

$|r| = \frac{1}{11} < 1$ so this is a convergent geometric series with first term $a = \frac{3}{11^{72}}$

So the sum is $\frac{a}{1-r} = \boxed{\dfrac{\frac{3}{11^{72}}}{1 + \frac{1}{11}}}$