MATH 231 U1, Spring 2009
Quiz 7, ANSWERS
Friday March 13th, 2009

1. Determine whether the following series converge absolutely, converge conditionally or diverge.

(a) $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{6^k}{(2k+1)!}$

ANSWER

Using the Ratio Test:

$$\lim_{k\to\infty}\frac{6^{k+1}}{(2k+3)!}\frac{(2k+1)!}{6^k}=\lim_{k\to\infty}\frac{6}{(2k+3)(2k+1)}=0<1$$

Since $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right|<1$, the series converges absolutely by the ratio test.

(b) $\sum_{k=2}^{\infty}(-1)^k\frac{1}{\ln k}$

ANSWER

Look at the series of absolute values $\sum_{k=2}^{\infty}\frac{1}{\ln k}$. We will apply the Limit Comparison Test with $\sum_{k=2}^{\infty}\frac{1}{k}$ which diverges by the p-test. (In class I did a regular Comparison Test, which works well too, and is shorter in this instance.)
Let $b_k=\frac{1}{k}$ and we know $a_k=\frac{1}{\ln k}$. Then

$$\lim_{k\to\infty}\frac{a_k}{b_k}=\lim_{k\to\infty}\frac{1}{\ln k}\frac{k}{1}=\lim_{k\to\infty}\frac{k}{\ln k}$$

Since

$$\lim_{x\to\infty}\frac{x}{\ln x}=\lim_{x\to\infty}\frac{1}{1/x}=\lim_{x\to\infty}x=\infty$$

By L'Hopital's Rule, we know that $\lim_{k\to\infty}\frac{k}{\ln k}=\infty$ too. Therefore, by the (expanded version) of the LCT (see problems 37 and 38 in 8.3) we know that $\sum_{k=2}^{\infty}\frac{1}{\ln k}$ diverges too.

So, our original series does NOT converge absolutely.

Now, $\lim k\to\infty\frac{1}{\ln k}=0$, and also $0<\frac{1}{\ln(k+1)}\leq\frac{1}{\ln k}$ because $\ln(k+1)\geq\ln k$. So, the series $\sum_{k=2}^{\infty}(-1)^k\frac{1}{\ln k}$ converges by the AST. Thus, it converges conditionally.
(Here the Ratio Test will be inconclusive, try it and see why!)

IMPORTANT NOTE: I sometimes use lots of words in my answer keys to try and help explain the answers clearly to you. You DO need to use English in your answers, but you don't always need to be as wordy as me. Just be sure you are:

- naming whatever tests you are using
- showing the hypotheses of these tests and of any theorems you are using are true
- stating your conclusions clearly

2. Find an upper bound for the error in approximating the series $\sum_{k=1}^{\infty}(-1)^k \frac{k}{2^k}$ by S_{100}.

Be sure to justify your answer.
(You DO NOT need to simplify)

ANSWER

Since I told you in class to assume this series converges by the AST, then by the Error Estimate for the AST, we know

$$|S - S_n| \leq a_{n+1}$$

Therefore

$$|S - S_{100}| \leq a_{101} = \frac{101}{2^{101}}.$$

Done.

If I had not told you to assume the series converges, you would need to show that the series satisfies the hypotheses of the AST in order to use this Error Estimate.