MATH 231 U1, Spring 2009
Quiz 8, Version 1
Friday April 3rd, 2009

ANSWERS

1. Find the interval of convergence for the power series $\sum_{k=0}^{\infty}(-2)^k x^k$. To what rational function does this power series converge?

$\sum_{K=0}^{\infty}(-2)^K X^K$ is a geometric series w/ ratio $r=(-2X)$

So it converges if $|r|<1$, i.e. if

$|-2X|<1$

$|2X|<1$

$|X|<\frac{1}{2}$ So its $\boxed{\text{R.O.C. is } \frac{1}{2}}$

$\boxed{\text{The series converges to } \frac{a}{1-r}=\frac{1}{1+2X} \text{ since it is geometric.}}$

a geom. series diverges if $|r|\geq 1$,

So, if $|-2X|=1$, this series diverges. Therefore it diverges when $X=\pm\frac{1}{2}$

So the $\boxed{\text{I.O.C. is } \left(-\frac{1}{2},\frac{1}{2}\right)}$

(NOTE: for a geom. series you never need to check the endpoints of the I.O.C., they will never be included. For a geom. series, once you have found the R.O.C. $=R$ you may just write down I.O.C. $=(C-R, C+R)$)

2. Find the power series representation centered at 0 for the function $g(x) = \ln(1 + 2x)$ and find its radius and interval of convergence. To help with this, note that $g'(x) = 2\frac{1}{1+2x}$, and therefore $2\int \frac{1}{1+2x}\,dx = g(x) + C$.

From Prob. 1: $\frac{1}{1+2x} = \sum_{k=0}^{\infty} (-2)^k x^k$

so $2\frac{1}{1+2x} = 2\sum_{k=0}^{\infty} (-2)^k x^k$

So $\underbrace{2\int \frac{1}{1+2x}\,dx}_{g(x)+C} = 2\int \sum_{k=0}^{\infty} (-2)^k x^k\,dx$

$= 2\sum_{k=0}^{\infty}\left[\int (-2)^k x^k\,dx\right] = 2\sum_{k=0}^{\infty} (-2)^k \frac{x^{k+1}}{k+1}$

So $\ln(1+2x) + C = 2\sum_{k=0}^{\infty} (-2)^k \frac{x^{k+1}}{k+1}$

plug in $x=0$ to find C: $\ln(1) + C = 2\sum_{k=0}^{\infty} 0 = 0$

$0 + C = 0$

$C = 0$

$$\boxed{\text{So } \ln(1+2x) = 2\sum_{k=0}^{\infty} (-2)^k \frac{x^{k+1}}{k+1}}$$

The R.O.C. is still $\frac{1}{2}$. We must check endpts:

If $x = \frac{1}{2}$ the series is $2\sum_{k=0}^{\infty} (-2)^k \frac{(\frac{1}{2})^{k+1}}{k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$ which converges by the AST.

If $x = \frac{-1}{2}$ $\ln(1+2x) = \ln(0)$ DNE!! So the series can't possibly converge at $x = -\frac{1}{2}$.

Thus the $\boxed{\text{I.O.C. is } (-\frac{1}{2}, \frac{1}{2}]}$