Math 103 Lecture 1 notes

Types of Problems:
1. **Process**: requires solution processes other than computational procedures. When attempting to solve a process problem, one uses available knowledge and employs strategies to devise a solution.
   Example:
   At an air show, 8 skydivers were released from a plane. Each skydiver was connected to each of the other skydivers with a separate piece of ribbon. How many pieces of ribbon were used in the skydiving act?
2. **Translation**: include one- and two-step story problems typically found in textbooks. These can be a medium for children to develop understanding of operations; i.e., they can enhance mathematical thinking. However, if presented after children have learned to compute, the problems are mere practice exercises and do not help to develop problem-solving ability.
   Example:
   A school auditorium can seat 648 people in 18 equal rows. How many seats are there in each row?
3. **Application**: computation is generally the solution process used to solve application problems. A calculator can lesson the time needed to arrive at an answer. Solving problems of interest to children can enhance their appreciation of mathematics.
   Example:
   How many soda cans would it take to fill the school gym?
4. **Puzzles**: solving puzzles may not require any mathematical knowledge, but are classified as a type of mathematical problem. It is often difficult to identify what strategy to use to solve a given puzzle. Usually, processes such as visualization, analysis, conjecturing, and testing are involved.
   Example:
   Can you join all nine dots using four straight lines without lifting your pencil from the paper?

   •   •   •
   •   •   •
   •   •   •
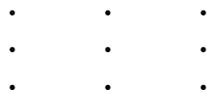
What is problem-solving?
"Problem-solving is the process of confronting a novel situation, formulating connections between given facts, identifying the goal of the problem, and exploring possible strategies for reaching the goal." (Szetela & Nicol, 1992) It requires the problem-solver to coordinate "previous experience, knowledge, and intuition" (Schoenfeld, 1989) in order to solve problems where no direct procedures or paths to the solution have been provided.

George Polya, a late Stanford University mathematician, described the experience of problem solving in his book, _How To Solve It_
 "A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings

into play your inventive facilities, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."

George Polya's 4-steps to prloblem-solving:

1.   Understand the problem
2.   Devise a plan (or strategy)
3.   Carry out the plan
4.   Look back

Problem-Solving Strategies

Break into smaller problems

Change your point of view

> Example: How can you cut a two-layer round cake into 8 pieces with exactly 3 cuts?

> Example: In the following cryptarithm - i.e., a collection of words where the letters represent numbers sun and fun represent two three-digit numbers and swim is their four-digit sum. Using all of the digits 0,1,2,3,6,7,9 in place of the letters where no letter represents two different digits, determine the value of each letter.

$$\begin{array}{r} \text{sun} \\ + \quad \text{fun} \\ \hline \text{swim} \end{array}$$

Classify

Combine strategies

Do a simulation

Dramatize the situation

> Example: Twelve people came to celebrate their ten-year high school reunion. Each person shook hands once with all the other persons. How many handshakes were exchanged at the reunion?

Draw a diagram

> Example: In a fraternity with 30 members, 18 take mathematics, 5 take both mathematics and biology, and 8 take neither mathermatics nor biology. How many take biology but not mathematics?

> Example: A pollster interviewed 500 University seniors who owned credit cards. She reported that 240 owned Goldcards, 290 had Supercards, and 240 had Thriftcards. Of those seniors, the report said that 80 owned only a Goldcard and a Supercard, 70 ownded only a Goldcard and a Thriftcard, 60 owned only a Supercard and a Thriftcard, and 50 owned all three cards. When the report was submitted for publication in the local campus newspaper, the editor refused to publish it, claiming the poll was not accurate. Was the editor right?

Draw a picture

> Example: Kyle and Jason watched the children's bicycle and tricycle parade during the summer festival. They agreed to keep count of the number of cycles and the number of wheels as the children'' parade passed by. At the end of the parade, Kyle declared he had counted 17 cycles and Jason said he had counted 43 wheels. How many bicycles and tricycles were in the parade?

> Example: While visiting a farm, you notice that there are only chickens and rabbits in the farmyard. You can't help but wonder how many of each animal there is in the yard. But when you ask Farmer Fred how many of each animal he has, he refuses to

give you a direct answer. (Some say he must have been a math teacher before he became a farmer.) Farmer Fred says that he will not tell you ow many of each kind of animal there is in the yard, but he will give you the information in another form. He tells you that there are 18 animal heads and 58 animal feet in the yard. Assuing that there are not mutant animals, how many chickens and how many rabbits must be in the yard?

Eliminate possibilities

Guess and test (or guess and check)

Example: Place the digits 1, 2, 3, 4, 5, and 6 in the circles, using each digit only once, so that the 3 numbers on each arm of the triangle add up to 12.
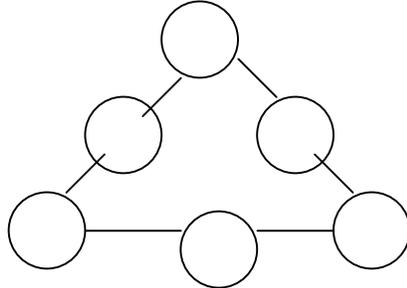
Identify sub-goals

Logical Reasoning

Example: Sudoku puzzles

Example: Aristotle has a 5-quart pail and a 3-quart pail and lots of water. Complete the descriptions of how he can measure out exactly 2 quarts of water using only these two pails and making no markings on the pails.

a) First fill the 3-quart pail.

b) First fill the 5-quart pail.

Look for a formula

Look for a pattern

Example: Imagine that you have been hired to decorate a room with streamers. The streamers are to be attached as the ceiling in such a way that all opposite corners of the room will be connected. How many streamers are nneded to decorate a 10-sided room?

Example: What is the next letter in this sequence?

O   T   T   F   F   S   S

Make a chart

Make a graph

Make a list

Make a scale drawing

Make a table

Solve a related problem

Solve a simpler problem

Solve an equivalent problem

Take a sample

Use a model

Use a variable

Use a variable

Use cases
Use coordinates
Use dimensional analysis
Use direct reasoning
Use examples
Use indirect reasoning
Use objects
Use properties of numbers
Use properties of numbers
Use symmetry
Work backwards
Write an equation

One way to help children become comfortable with the problem-solving process is to have children write problems. Through such experiences, children develop critical thinking, learn to collect and organize data, and learn how to express their ideas in a clear and succinct manner.

The activity of posing problems is different from that of writing problems. Posing problems usually refers to the process of changing and existing problem into a new one by modifying the *knowns*, the *unkowns*, or the restrictions placed on the answer.

Four principles for guiding children to engage in posing problems:
1. Have children learn to focus their attention on known, unknown, and restrictions.
2. Begin with a comfortable mathematics topic.
3. Encourage children to use ambiguity to create new questions and problems.
4. Teach the idea of domain from the earliest grades, encouraging children to "play the same game with a different set of pieces."

In designing or selecting math problems, one goal should be to maximize the potential of the precious little time each classroom spends on math. While there are many potentially "good" problems, there are also many levels at which a problem is or is not "good."
5. If students love the activity but do not learn any math from it, it is not good.
6. If the students are engaged in math, but the activity is not manageable in the classroom, then it is not good.
7. If the potential math is superb, but students aren't interested in the activity, then it is not good.

Which of the following is a good mathematical problem?
a. 42 x 37 = ?
b. Nora has 42 people coming to her party. She wants to give each of them a bag of 37 peanuts. How many peanuts does she need to buy?
c. Each year the fifth grade takes a trip to Washington, DC. They have a car wash to raise money to pay for their expenses. This year there are 42 students in the fifth grade. The

bus fare and lunch will cost $37 per student. How much money do they need to raise to pay for the expenses of all the students?

d. None of the above.

e. All of the above.

George Polya: Father of Modern Problem-solving
"Ten-Commandments for Teachers"

1. Be interested in your subject
2. Know your subject
3. Try to read the faces of your students; try to see their expectations and difficulties; put yourself in their place.
4. Realize that the best way to learn anything is to discover it by yourself.
5. Give your students not only information, but also know-how, mental attitudes, and the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving problems to come - try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once - let the students guess before you tell it - let them find out by themselves as much as if feasible.
10. Suggest it; do not force it down their throats.

Every weekday, 25 million children study mathematics in our nation's schools. The 40 classroom minutes they spend on mathematics each day are largely devoted to mastery of the computational skills which would have been needed by a shopkeeper in the year 1940 . . . skills needed by virtually no one today. Almost n time is spent on estimation, probability, interest, histograms, spreadsheets, or real problem-solving . . . things which will be commonplace in most these young people's later lives. While the 15 million of them sit there drilling away on those arithmetic or algebra exercises, their future options are bit-by-bit eroded. (Mathematical Sciences Education Board, 1986)

Virtually all young children like mathematics. They do mathematics naturally, discovering patterns and making conjectures based on observation. Natural curiosity is a powerful teacher, especially for mathematics.

Unfortunately, as children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory, Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. (Everybody Counts p. 43)

Lab #1: The goal for lab #1 is to understand how the State Board (ISBE) assesses a student's ability to solve problems.   You will be trained to use to ISAT Rubric to assess samples of student responses to open-ended problems.