Section4.3 Exercise7:
Since A is similar to B, there exists a nonsingular matrix S such that $A = S^{-1}BS$. Also since B is similar to C, there is a nonsingular matrix T such that $B = T^{-1}CT$. Then we know TS is nonsingular and $A = S^{-1}(T^{-1}CT)S = (TS)^{-1}C(TS)$. Thus A is similar to C. $\square$

Section4.3 Exercise10:
Since A is similar to B, there exists a nonsingular matrix S such that $A = S^{-1}BS$. Now let $T = BS^{-1}$, we then have $A = ST$ and $B = TS$ as required. $\square$

Section5.1 Exercise11:
a) We have $\mathbf{x}^T\mathbf{x} = (x_1^2 + X_2^2) \geq 0$.
b) We have $\mathbf{x}^T\mathbf{y} = x_1y_1 + x_2y_2 = y_1x_1 + y_2x_2 = \mathbf{y}^T\mathbf{x}$.
c) We have $\mathbf{x}^T(\mathbf{y}+\mathbf{z}) = \mathbf{x}^T(y_1 + z_1, y_2 + z_2) = x_1(y_1 + z_1) + x_2(y_2 + z_2) = x_1y_1 + x_2y_2 + x_1z_1 + x_2z_2 = \mathbf{x}^T\mathbf{y} + \mathbf{x}^T\mathbf{z}$. $\square$

Section5.1 Exercise14:
a) We know $\alpha^2 + h^2 = \|\mathbf{a_1}\|^2$ and $\alpha = \frac{\mathbf{a_1}^T\mathbf{a_2}}{\|\mathbf{a_2}\|}$. Using these two identities we get $h^2\|\mathbf{a_2}\|^2 = (\|\mathbf{a_1}\|^2 - (\frac{\mathbf{a_1}^T\mathbf{a_2}}{\|\mathbf{a_2}\|})^2)\|\mathbf{a_2}\|^2 = \|\mathbf{a_1}\|^2\|\mathbf{a_2}\|^2 - (\mathbf{a_1}^T\mathbf{a_2})^2$.
b) Write $\mathbf{a_1} = (a_{11}, a_{12})$ and $\mathbf{a_2} = (a_{21}, a_{22})$. We get $h^2\|\mathbf{a_2}\|^2 = (a_{11}^2 + a_{12}^2)(a_{21}^2 + a_{22}^2) - (a_{11}a_{21} + a_{12}a_{22})^2 = (a_{11}a_{22} - a_{21}a_{12})^2 = |det(A)|^2$. Hence Area P $= h\|\mathbf{a_2}\| = |det(A)|$. $\square$

Section5.2 Exercise6:
It is impossible. If such matrix A exists, then we know $(3,1,2)^T$ is in column space of A^T, which is $R(A^T)$ and we have $(2,1,1)^T \in N(A)$. However these two subspaces are orthogonal to each other, but

$$< (3,1,2)^T, (2,1,1)^T >= 9 \neq 0.$$

$\square$

Section5.2 Exercise16:
First we recall from page 232 that $R(A) = \{A\mathbf{y} | y \in R(A^T)\}$, since $\{\mathbf{x_1}, ..., \mathbf{x_r}\}$ spans $R(A^T)$ we know $\{A\mathbf{x_1}, ..., A\mathbf{x_r}\}$ spans $R(A)$. Now we have to prove that $\{A\mathbf{x_1}, ..., A\mathbf{x_r}\}$ is linear independent. Let $0 = \sum_{i=1}^r c_iA\mathbf{x_i} = A(\sum c_i\mathbf{x_i})$ this shows $\sum c_i\mathbf{x_i} \in N(A)$. Also we know $\sum c_i\mathbf{x_i} \in R(A^T)$ because each $\mathbf{x_i}$ is in $R(A^T)$. Thus we know $\sum c_i\mathbf{x_i} \in N(A) \bigcap R(A^T) = \{0\}$, that is $\sum c_i\mathbf{x_i} = 0$. Which implies $c_i = 0$ for all i since $\{x_i\}$ is linear independent. This shows that $\{A\mathbf{x_1}, ..., A\mathbf{x_r}\}$ is linear independent. $\square$