

Differentiable Manifolds: Problem set 1

Due Wednesday September 3

Read the handout on topology and report any typos to me (due whenever!). Work the exercises if they are unfamiliar (you need not hand these in) and feel free to consult a point set topology text if necessary.

Read Guillemin and Pollack (GP for short) Chapter I section 1 and 2. Do problems §1.1: 5, 7, 12, 13, 18. (Note that in GP, “manifold” actually means more than topological manifold. Use our definition of smooth manifold and smooth function/map for these when necessary).

Read Warner p 1–11.

Additional problem.

(1.) Suppose that $M \subset \mathbb{R}^d$ is a smooth n -manifold in the sense of GP. Construct an atlas on M so that a function $f : M \rightarrow \mathbb{R}$ is smooth in the sense of GP if and only if it is smooth in our sense. Similarly, if we also have $N \subset \mathbb{R}^k$, a smooth k -manifold, prove that a smooth map $f : M \rightarrow N$ is smooth in the sense of GP if and only if it is smooth in our sense.

The purpose of this exercise is to show that our notion of smooth manifold is compatible with, and at least as general as, the one given in GP. Soon we will show that every smooth manifold in our sense can also be obtained as in GP.

Notational suggestion for this problem: define “GP-smooth” functions to be those which are smooth in the sense of GP, and reserve “smooth” to mean smooth in our sense. After this exercise, there will be no further need to make this distinction.

General comment on homework: You do not need to copy the problems down. Just give the location (GP, Warner or handout), and the number. Write in complete sentences.