Differential Geometry: Problem set 1

August 23, 2006

Due Wednesday August 30

Read the handout on topology and report any typos to me (due whenever!). Work the exercises if they are unfamiliar (you need not hand these in) and consult a point set topology text if you need to.

Boothby: Read Chapter I (this is casual reading) and Chapter II sections 1 and 2.

Read Guillemin and Pollack (GP for short) Chapter I sections 1 and 2. Do problems §1.1: 5, 7, 14, 18. §1.2: 5. (use our definition of manifold and smooth function/map for these when necessary).

(1.) Following the suggestion of GP §1.1 problems 12 and 13, define an atlas on the n-sphere, S^n, for $n \geq 1$ using two sets of stereographic coordinates—one defined on the complement of the north pole $N = (0, 0, ..., 0, 1)$ and one defined on the complement of the south pole $S = (0, 0, ..., 0, -1)$. Show that your two coordinate charts define an atlas (verify all the conditions for an atlas). Check that this defines the same smooth structure as the atlas we defined in class.

(2.) Suppose that $M \subset \mathbb{R}^N$ is a smooth n-manifold in the sense of GP. Construct an atlas on M so that a function $f : M \to \mathbb{R}$ is smooth in the sense of GP if and only if it is smooth in our sense. Similarly, prove that a smooth map $f : M \to N$ is smooth in the sense of GP if and only if it is smooth in our sense.

The purpose of this exercise is to show that our notion of manifold is compatible with, and at least as general as the one given in GP. We'll soon see that in some sense it is actually no more general.

Notational suggestion for this problem: define "GP-smooth" functions to be those which are smooth in the sense of GP, and reserve "smooth" to mean smooth in our sense.

General comment on homework: You do not need to copy the problems down. Just give the location (Boothby, GP, or handout), and the number. Write in complete sentences.