Math 231: Honors Question 1

Due Wednesday, January 31

Consider a function $f(x)$ defined on the interval $[-\pi, \pi]$. Prove that if f' is continuous on $[-\pi, \pi]$, then the Fourier coefficient

$$a_n = \int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

tends to zero as n tends to infinity. That is, prove

$$\lim_{n\to\infty} a_n = 0$$

The precise meaning of this limit is that for every $\epsilon > 0$, there exists an integer N so that whenever $n \geq N$, we have

$$|a_n| < \epsilon$$

Hint: Use integration by parts. The following fact might also be useful:

$$\left|\int_a^b k(x)dx\right| \leq \int_a^b |k(x)|dx$$

for any integrable function $k(x)$*.*