We now switch gears, turning our attention to topology. In particular, the space $\mathbb{R}^3 \setminus L$ (or $S^3 \setminus L$) when $L$ is a link.

If $L \sim L'$, then we have a homeomorphism $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ with $f(L) = L'$.

This $f$ restricts to a homeomorphism $\hat{f} : \mathbb{R}^3 \setminus L \rightarrow \mathbb{R}^3 \setminus L'$.

Therefore, quantities depending only on the homeomorphism type of $\mathbb{R}^3 \setminus L$ become invariants of $L$.

A deep result of the late 1980's by Gordon and Luecke is that if $K \sim K'$ are knots and $\mathbb{R}^3 \setminus K$ and $\mathbb{R}^3 \setminus K'$ are homeomorphic, then either $K \sim K'$ or $K \sim \overline{K'}$.

False for links:

- and  

have homeomorphic complements.

To see this "cut open" along disk, twist, and reglue

more later on this.