

## Midterm 2 Rough outline

Important: You must present your ID when you turn in your exam!! Your exam will be **REFUSED** if you do not have your ID on test day!!!!

For the exam, you should be able to:

- Represent vectors in terms of a basis and be able to change bases.
- Find bases for the row and column space of a matrix.
- Prove basic facts about matrices based on the relation between rank and nullity, and null space, row space, and column space.
- Decide consistency of a system based on the column space.
- Decide whether or not a function/mapping  $T : V \rightarrow W$  between vector spaces  $V$  and  $W$  is a linear transformation.
- Prove basic facts about linear transformations, their kernels and their images.
- Find matrices representing a linear transformation with respect to bases, and be able to change bases.
- Prove basic facts about linear transformations using matrix representations.
- Prove basic facts about similar matrices.
- Compute and understand the meaning of the scalar product in  $\mathbb{R}^n$ .
- Compute vector projections.
- Prove basic facts about the scalar product (and more generally, inner products) and orthogonality.
- Prove basic facts about subspaces, direct sum and orthogonal projections.
- Understand the relationship between column space and row space for matrices and their transposes—especially the orthogonality relations.
- Solve least squares problems, and understand the idea of the least squares problem and the nature of its solution.
- Decide whether or not sets are orthogonal/orthonormal sets.
- Do computations and proofs in inner product spaces using orthogonal/orthonormal bases and orthogonal projections.
- Prove basic facts about orthogonal matrices.
- Apply Gram-Schmidt Process to produce orthonormal bases and find  $QR$  factorization of maximal rank matrices.