Name: _____

Instructions: Do every problem. For full credit, be sure to show all your work. The point is to show me that you know HOW to do the problems, not that you can get the right answer, possibly by accident.

1. There will be 8 true false questions regarding: geometry of vectors in 3-space (dot products, cross products, scalar triple products); the definition of continuity, differentiation (and applications), and integration (in all its guises...); Green's Theorem, Divergence Theorem, Stoke's Theorem, Conservative vector fields.

2. Consider the graph of the function $y = x^2$ as a curve and compute the curvature at $(0, 0)$.

3. There will be a matching section: matching surfaces with the equations that define them. You will not be able to use the process of elimination....

Here are a few equations, sketch the level surfaces they define (you will not need to sketch them on the exam, just match them).

$$x^2 - y^2 - z^2 = 1 \qquad\qquad x^2 + \frac{y^2}{4} - z = 2$$
$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1 \qquad\qquad x^2 + \frac{z^2}{9} - y^2 = 1$$

4. Find the critical points of the function $f(x, y) = x^2 + y^2 + xy + x - y$.

5. Find the tangent plane to the level surface $x^2 + y^2 - z^2 = 1$ at the point $(1, 1, 1)$.

6. Compute the integral

$$\iint_D xy \, dxdy$$

where D is the region bounded by the ellipse with defining equation $x^2 + 2y^2 - 2xy = 1$. **Hint: Consider the change of variables $T(u, v) = (u + v, v)$.**

7. Let $\mathbf{F} = \langle xyz, y^2, -x^2 \rangle$ and $f(x, y, z) = xy - 1$. Compute each the following:

a. $\nabla \times \mathbf{F}$

b. $\nabla \cdot \mathbf{F}$

c. $(\nabla f) \cdot \mathbf{F}$

d. $\nabla \times (f\mathbf{F})$.

8. Calculate

$$\int_C (x^2 + y^2 - y)\ dx\,(y^2 - 2x^3)\ dy$$

where C is the positively oriented boundary of the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

9. Consider the unit box B defined by $0 \leq x \leq 1$, $0 \leq y \leq 1$, and $0 \leq z \leq 1$. Let S be the surface which is the portion of the boundary of B consisting of the top and four sides (that is, the boundary less the bottom) and let

$$\mathbf{F} = \langle y^2 + xz^{10} - e^{xz}, 3xy^2 + eyz, xyz + \cos(xy) \rangle$$

Evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA_S$$