Final Exam Review Sheet

For the final exam, you should:

- Be able to compute indefinite integrals using the techniques of substitution and integration by parts.
- Be able to compute integrals of trigonometric funtions, especially products of such.
- Be able to use trig substitutions to compute integrals.
- Be able to compute integrals of rational functions using partial fraction decompositions.
- Know what an improper integral is and what it means for such to converge.
- Be able to evaluate improper integrals.
- Be able to solve basic problems involving exponential growth and decay.
- Understand the notion of convergence for sequences and series and what the relationship is between the two.
- Be able to use all the tests for series to decide convergence or divergence. Specifically, you should be comfortable using the following tests for series and know when they can and can't be used:
  - the n^{th} term test for divergence,
  - the integral test,
  - the comparison and limit comparison tests,
  - the alternating series test,
  - the ratio and root tests.
- Be able to decide whether a convergent series converges absolutely or conditionally (and you should know what this means).
- Know some basic series and their convergence properties (e.g. p-series and geomtric series) to use in comparisons.
- Know what a power series is and what it's radius and interval of convergence are.
- Be able to compute the radius and interval of convergence for a power series.
- Be able to maniputе power series algebraically and using calculus.
- Know what Taylor and Maclaurin series are and different ways of finding them for a function (including the definition!).
- Know what Taylor's Theorem says, and how it can be used to prove convergence of a power series to a function.
- Know how to use power series to make estimates.
- **DON'T FORGET INDETERMINANT FORMS AND L'HOPITAL'S RULE!**
- Be able to parameterize "basic curves".
- Be able to sketch a simple parameterized curve (with a focus on qualitative features).
- Know how to find the slope of the tangent line to a planar curve using a parameterization.
- For a planar curve, use parameterized curves to be able to compute arc length and the surface area for surfaces of revolution.
- Be able to compute the area bounded by a closed planar curve.
- Understand the basic relationship between polar and rectangular coordinates and be able to convert between the two.
- Be able to sketch a polar curve.
- Be able to find the tangent line and arc length of a polar curve.
- Be able to find the area bounded by a polar curve and between two polar curves.

The following will be given to you with the final exam:

A few integrals

$$\int \sin x\,dx = -\cos x + c \qquad \int \cos x\,dx = \sin x + c \qquad \int \sec^2 x\,dx = \tan x + c$$

$$\int \sec x \tan x\,dx = \sec x + c \qquad \int \csc^2 x\,dx = -\cot x + c \qquad \int \csc x \cot x\,dx = -\csc x + c$$

$$\int e^x\,dx = e^x + c \qquad \int \tan x\,dx = -\ln|\cos x| + c \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$$

A few power series

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!}x^k \qquad \ln x = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}(x-1)^k \qquad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k} \qquad \tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}x^{2k+1}$$

A parameterized curve

$x = x(t)$, $y = y(t)$, $a \le t \le b$

Speed $\qquad s(t) = \sqrt{(x'(t))^2 + (y'(t))^2}$

Slope of tangent line $\qquad m(t) = \dfrac{y'(t)}{x'(t)}$

Arc length $\qquad L = \int_a^b s(t)dt$

Area for surface of revoluation $\qquad A = \int_a^b 2\pi(\text{distance to axis})s(t)dt$

Area bounded by curve (assumed closed, traversed once clockwise) $\qquad A = \int_a^b y(t)x'(t)dt = -\int_a^b x(t)y'(t)dt$

Polar curves

$r = f(\theta)$ (and $r = g(\theta)$)

$\theta_1 \le \theta \le \theta_2$

Slope of tangent line $\qquad m(\theta) = \dfrac{f'(\theta)\sin(\theta) + f(\theta)\cos(\theta)}{f'(\theta)\cos(\theta) - f(\theta)\sin(\theta)}$

Arc length $\qquad \int_{\theta_1}^{\theta_2} \sqrt{(f'(\theta))^2 + (f(\theta))^2}d\theta$

Area bounded by $r = f(\theta)$ $\qquad \int_{\theta_1}^{\theta_2} \frac{1}{2}(f(\theta))^2 d\theta$

Area between curves $\qquad \int_{\theta_1}^{\theta_2} \frac{1}{2}|(f(\theta))^2 - (g(\theta))^2| d\theta$