TESTS FOR SERIES

Series

$$\sum_{n=1}^{\infty} a_n \quad \& \quad \sum_{n=1}^{\infty} b_n$$

Test	tests for	how it works
n^{th} term test	divergence only	series diverges if $\lim a_n \neq 0$ or limit does not exist.
Integral test	convergence or divergence	for positive term series with $a_n = f(n)$ where $f(x)$ is a positive nonincreasing function, the series converges if and only if the improper integral $\int_1^\infty f(x)dx$ converges
Comparison test	convergence or divergence	for two positive term series with $a_n \geq b_n$ If $\sum a_n$ converges then $\sum b_n$ converges If $\sum b_n$ diverges then $\sum a_n$ diverges.
Limit comparison test	convergence or divergence	for two positive term series with $\lim \frac{a_n}{b_n} = L$ $0 < L < \infty$, then $\sum a_n$ converges if and only if $\sum b_n$ converges.
Ratio test	convergence or divergence	for positive term series if $\lim \frac{a_{n+1}}{a_n} = L$ exists then $\sum a_n$ converges if $L < 1$ and diverges if $L > 1$. No info if $L = 1$.
Root test	convergence or divergence	for positive term series if $\lim \sqrt[n]{a_n} = L$ exists then $\sum a_n$ converges if $L < 1$ and diverges if $L > 1$. No info if $L = 1$.
Alternating series test	convergence only	for alternating series: $a_n \geq 0$, the series $\sum (-1)^{n+1} a_n$ is an alternating series. This converges if $a_n \geq a_{n+1}$ and $\lim a_n = 0$.

Good series to know the behavior of: $\sum \frac{1}{n^p}$ p–series and $\sum ar^n$ geometric series.

Two mutually disjoint possibilities for an *arbitrary* series $\sum a_n$: convergence or divergence.

Two mutually disjoint types of convergence:
absolute convergence: $\sum |a_n|$ converges (which implies $\sum a_n$ converges).
conditional convergence: $\sum a_n$ converges **and** $\sum |a_n|$ diverges.

Any test for positive term series can be used as a test for absolute convergence since $\sum |a_n|$ is a positive term series:

A caveat: If we use one of the tests for positive term series to test for absolute convergence, and it fails, we cannot decide that the series itself diverges (it could be that it converges conditionally). The ratio and root tests are exceptions: If the limit of $\frac{|a_{n+1}|}{|a_n|}$ is $L > 1$, then the series does indeed diverge.