HOMEWORK 1

Problem 1

Prove that the composition of two bijections is again a bijection.

Problem 2

Prove directly from the definitions that the union of two countable sets is countable. Feel free to use without proof the fact that the union of two finite sets is finite.

Problem 3

Prove that the set of all finite subsets of $\mathbb{N}$ is countable.

Problem 4

Prove DeMorgan's Laws: that is, prove that $(A \cup B)^C = A^C \cap B^C$ and that $(A \cap B)^C = A^C \cup B^C$

Problem 5

Here's a puzzle to let you practice your mathematical induction skills. Consider the following game: There are two piles of coins, and two players. The players alternate turns, and on each turn a player can take any positive number of coins from one of the two piles. The player that takes the last coin wins. The two piles begin with the same number of coins. Does one of the players have a winning strategy? Prove it.

Problem 6

Prove that each subset of a countable set is countable.