# HOMEWORK 10

Due Friday, April 29th

## Problem 1

Initially, this question was going to be on the exam, but then I decided that the exam was long enough without it. So now it's homework:

**(a).** Fix a model $\mathfrak{M}$, let $T$ be $Th(\mathfrak{M})$. Pick an element, $m$, of the model. Show that the set of all formulas that hold of $m$ form a type.

**(b).** Now let $\mathfrak{M}$ be $(\mathbb{N}, +, \cdot, <, 0, 1)$. Is every type realized by some element of $\mathbb{N}$? (An element $n$ *realizes* a type iff every formula in the type is true of $n$).

**(c).** Keeping $T$ and $L$ as in part (b), is every type consisting of only finitely many formulas realized in $\mathbb{N}$?

**(d).** Again keeping $T$ and $L$ as in part (b), is every type realized in some model $\mathfrak{N}$ that is elementarily equivalent to $\mathbb{N}$?

## Problem 2

Show that for any language $L$ complicated enough that the valid sentences in $L$ are undecidable, that the satisfiable sentences in $L$ are not even enumerable.

## Problem 3

Let $\mathfrak{N} = (\mathbb{N}, +, \cdot, 0, 1, <)$. Show that any set defined by a formula, $\psi(z)$, of the form $\exists x_1, \ldots, \exists x_n \varphi(x_1, \ldots, x_n, z)$ where $\varphi$ has no quantifiers, is enumerable.