HOMEWORK 2

Problem 1

Here is a lemma that I mentioned should be proven in order for cardinality to behave as it should: Let A and B be sets. Prove that there is a surjection from A to B iff there is an injection from B to A.

Problem 2

Prove that the set of bijections from $\mathbb{N}$ to $\mathbb{N}$ is uncountable.

Problem 3

Prove that $|[0,1]| = |\mathbb{R}|$.

Problem 4

Prove that every formula in propositional logic has an even number of parentheses.

Problem 5

Which of the following are tautologies? Prove.

(a). $(p \to q) \leftrightarrow (q \to p)$

(b). $(p \to q) \leftrightarrow (\neg q \to \neg p)$

(c). $(p \wedge q) \to (p \vee q)$

Problem 6

A king has a prisoner locked up in a tower. The tower has two exits, one of which leads to freedom, the other into the torture chamber. A guard is standing in front of each exit. One of the guards always tells the truth, the other always lies. The guards each know what is behind the exits, and they know about each other's (un)truthfulness. The prisoner knows that one guard tells the truth and the other lies, but doesn't know which is which, and neither does he know which door leads to freedom. The king gives each prisoner one chance to ask one of the guards a single question which can be answered either "yes" or "no", in order to find the door to freedom. What should the prisoner ask?

For more of these puzzles, and an explanation as to why they might have a relevance to mathematical logic, look up the books of Raymond Smullyan.