HOMEWORK 3

Problem 1

Let ϕ and ψ be formulas in propositional logic. Show that $\phi \vee \psi$ is equivalent to $\neg(\neg\phi \wedge \neg\psi)$.

Problem 2

In Problem 1, you showed that "or" can be expressed in terms of "and" and "not". Can you do the same with "implies"? That is, given a formula $\phi \to \psi$ in propositional logic, can you find an equivalent formula using ϕ, ψ, $\wedge$, and $\neg$? What about "if and only if"?

Problem 3

Prove that any formula in propositional logic is equivalent to a formula using only $\wedge$ and $\neg$.

Problem 4

Fix an alphabet A, and suppose that A is countable. That is, there are a countable number of relation, function, and constant symbols. Prove that there are countably many formula in first order logic using the alphabet A..

Problem 5

Note that the word "buffalo" is both noun (big cow-like creature that used wander the open plains in big herds) and a verb (either to intimidate with a display of power or to confuse). What's the longest English sentence you can make using only the word "buffalo"? Since this is a math course and shouldn't be testing your English language skills, I'll give a hint, but I'll wait until Monday to give people that want to think about it without a hint a chance to do so.

Problem 6

For each of the following formulas, write down (i) the list of subformulas, and (ii) the list of free variables.

(a). $\theta_1 := \forall x, y \exists z\, ((x < y) \to ((x < z) \wedge (z < y)))$

(b). $\theta_2 := \exists z(y + y < z)$

(b). $\theta_3 := \forall x((x < y) \vee (x = y)) \vee (x + x = x)$ (And, yes, that is how I want the parentheses.)