

HOMEWORK 5

PROBLEM 1

In this problem we return the language and theory described in Problem 5 of last week's homework. To remind you, L is a language consisting of a single unary function S , and T consists of the following sentences:

$$\sigma_1 := \forall x \neg (S(x) = x).$$

$$\sigma_2 := \forall x \neg (S(S(x)) = x), \text{ let } \sigma_3 := \forall x \neg (S(S(S(x))) = x), \text{ and so on, for each } n$$

$$\theta := \forall x \exists y (S(y) = x), \text{ and let } \psi := \forall x \forall y (S(x) = S(y) \rightarrow (x = y)).$$

Last week you showed that T was consistent by finding a model for T . This week, show that no finite subset of T implies all of T .

PROBLEM 2

Find a finite set of sentences of first order logic (in any language you like), such that the any model of the finite set of sentences is infinite.

PROBLEM 3

Another problem moving a bit closer to Gödel's incompleteness Theorem: A student travels to the island of knights and knaves to study with a noted philosopher, who is a knight. One day, in the midst of a lecture on the dangers of self-satisfaction and the hazards of pride, the philosopher says to the student, "You will find true happiness if and only if you do not correctly believe that you will find true happiness".

- (a). Show that the student may correctly deduce that he will find true happiness.
- (b). Show that if the student is a reasoner of type 1 (see solutions of last week's homework for a definition), then he will also be able to correctly deduce that he will not find true happiness, and hence he will fall into contradiction.