HOMEWORK 6

Problem 1

Define the equivalence relation $\sim$ on $\mathbb{Z}$ as follows: Let $a \sim b$ iff $a - b$ is divisible by 7.

(1). Show that this is indeed an equivalence relation.

(2). Define $+$ on $\mathbb{Z}/\sim$ as follows: Let x, y be two elements of $\mathbb{Z}/\sim$ (i.e. two equivalence classes of $\sim$ in $\mathbb{Z}$). Pick some $n \in x$ and $m \in y$. Define $x + y$ to be the equivalence class to which $m + n$ belongs. Prove that this function is well-defined. Give a similar definition for multiplication, and prove that this too is well-defined. (To all those of you who have done this before, I apologize if this is boring.)

Problem 2

Fix a language L, and let f be a unary function in the language. Show that if a finite collection of formulas, Γ proves $t_1 = t_2$ then Γ proves that $f(t_1) = f(t_2)$.

Problem 3

Suppose that T is a consistent set of sentences, and φ is any formula, then either $T \cup \{\varphi\}$ is consistent, or $T \cup \{\neg\varphi\}$ is consistent (or both, of course).