HOMEWORK 6

Problem 1

Pick a language L, and pick an inconsistent Φ in consisting of formulas from the language L. Construct $\mathfrak{T}$, the structure with universe equal to the set of terms modulo $\sim$, as in class. Does $\mathfrak{T}$ depend on T? Does $\mathfrak{T}$ model Φ?

Problem 2

Pick a language L and for each term in the language let φ_t be the formula $v_0 = t$. Let θ be the formula $\exists v_0 \exists v_1 (\neg(v_0 = v_1))$. Let T be the collection of all φ_t together with θ. Show that T is consistent, but there is no consistent set of L formulas containing T which contains witnesses.

Problem 3

Show that if T has arbitrarily large countable models, then T has an infinite model. Is there a theory T true of all finite groups, but not true of any infinite groups?

Problem 4

Let $L := \{+, \cdot, 0, 1, <\}$.

(a). Show that each rational number is definable in $(\mathbb{R}, +, \cdot, 0, 1, <)$. Note that I am not asking for you to show that the set of all rational numbers is a definable set, but rather each individual rational number is a definable element.

(b). Show that there is a model $\mathfrak{R}$ such that $\mathfrak{R}$ and $(\mathbb{R}, +, \cdot, 0, 1, <)$ satisfy precisely the same set of formulas, and such that there is an element of $\mathfrak{R}$ greater than zero and less than every positive rational number. (Such an element is called an *infinitessimal* element."