HOMEWORK 9

Problem 1

(a). First we consider the model $(\mathbb{Z}, +)$. Is 1 a definable element in $(\mathbb{Z}, +)$? (Give a proof, of course.) List all the definable elements of $(\mathbb{Z}, +)$.

(b). Now consider the model $(\mathbb{Z}, +, <)$. Prove that each element of $(\mathbb{Z}, +, <)$ is definable. List all of the automorphisms of $(\mathbb{Z}, +, <)$

Problem 2

Prove that there is a model $\mathfrak{Z}$ elementarily equivalent to $(\mathbb{Z}, +, <)$ where it is not the case that every element is definable.

Problem 3

Let $W \subseteq A^*$. Show that the following are equivalent:
(1) W is enumerable.
(2) There is register machine program such that P on input w outputs the empty word and halts whenever $w \in W$, and P runs forever without outputing anything if w in not in W.

Problem 4

A set $W \subseteq A^*$ is called lexiographically enumerable iff there is a register machince program that outputs W is lexiographic order. Show that W is lexiographically enumerable iff W is decidable.

Problem 5

Let $A := \{a, \}$ Carefully write out a register program that takes a string $w \in A^*$ and and halts iff the length of the string in divisible by three.

Problem 6

Suppose that $U \subseteq A^*$ is decidable, and suppose that $W \subseteq U$. Show that W is decidable if and only if both $U \setminus W$ and W are enumerable.