PROBLEM 1

(a). Let $f(x) = \frac{x^3-1}{x^2+2x-3}$. Does the limit of $f(x)$ as x approaches 3 exist? If so find the limit. Is $f(x)$ continuous at 3? If not, is the discontinuity removable?

Both x^3-1 and x^2+2x-3 are continuous and x^2+2x-3 is nonzero for $x=3$. So $f(x)$ is continuous at 3. Hence the limit exists and equals $f(3) = \frac{26}{12} = 13/6$

(b). Let $f(x)$ be as in part (a). Does the limit of $f(x)$ as x approaches 1 exist? If so find the limit. Is $f(x)$ continuous at 1? If not, is the discontinuity removable?

Note x^2+2x-3 is zero for $x=1$. Thus $f(x)$ is not continuous at $x=1$. Note that x^3-1 is also 0 at $x=1$. Thus, we factor, obtaining $\frac{(x-1)(x^2+x+1)}{(x-1)(x+3)} = f(x)$.

Thus $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2+x+1}{x+3} = 3/4$

(c). Let $g(x) = x \cot x$. Does the limit of $f(x)$ as x approaches 0 exist? If so, find the limit.

We rewrite $\cot x$ as $\frac{\cos x}{\sin x}$, obtaining $g(x) = \frac{x}{\sin x} \cos$

Thus $\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{x}{\sin x} \cdot \lim_{x\to 0} \cos x = 1 \cdot 1 = 1$

Problem 2

(a). Let $f(x) = (\frac{x+1}{1-x})^5$. Find $f'(x)$.

$$f'(x) = 5\left(\frac{x+1}{1-x}\right)^4 \cdot \frac{(1-x)+(x+1)}{(1-x)^2}$$

$$= \frac{10(x+1)^4}{(1-x)^6}$$

(b). Let $g(t) = \cos t + (\sec t)^3$. Find $g'(t)$.

$$g'(t) = -\sin t + 3(\sec t)^2 \sec t \tan t$$

or, if you prefer, $\sin t(3\sec^4 t - 1)$

(c). Let $h(y) = \csc(y^2+2)$. Find $h'(y)$.

$$h'(y) = \csc(y^2+2)\cot(y^2+2) \cdot 2y$$

PROBLEM 3

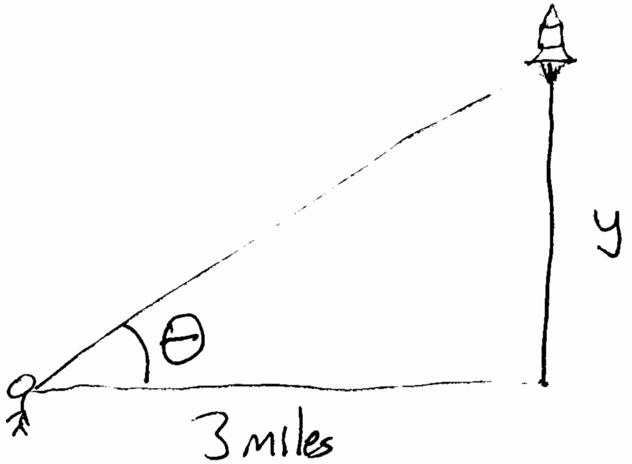
Consider the graph of $f(x) = x^2 + 3x + 2$. Find the equation of the tangent line to the graph with slope 5.

We note that $f'(x) = 2x+3$. Thus we want to find the point on the curve where f' is equal to 5. When $2x+3=5$ we have $x=1$, and ~~[illegible]~~ thus the point we want is $(1, f(1)) = (1,6)$

Using point-slope form, we see that equation we want is $(y-6) = 5(x-1)$

PROBLEM 4

A rocket is launched 3 miles away from an observer. The observer notes that when the angle of elevation is 30 degrees, the angle is increasing at 6 degrees per second. What is the vertical speed of the rocket?

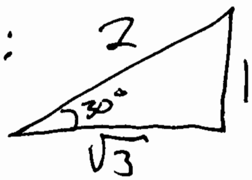


When the rocket is at an angle of elevation of θ from the observer it is at a height $y = 3\tan\theta$

So $\frac{dy}{dt} = 3\sec^2\theta \frac{d\theta}{dt}$.

We are told $d\theta/dt$ is 6 degrees/sec $= \frac{6\pi}{180}$ ~~rad~~/sec $= \pi/30$ rad/sec

So $\frac{dy}{dt} = 3\sec^2 30° \cdot \pi/30$ Recall:



So $\sec 30° = \sec \pi/6 = \frac{2}{\sqrt{3}}$

Thus $\frac{dy}{dt} = 3 \cdot 4/3 \cdot \pi/30 = 2\pi/15$ miles/sec.

PROBLEM 5

Find the two non-negative real numbers, whose sum is ten, and the sum of whose squares is as large as possible.

We have $x+y=10$ and we wish to maximize $s=x^2+y^2$. We note x and y may vary between 0 and 10 (inclusive). And we note that $y=10-x$, so we want to maximize $s(x)=x^2+(10-x)^2$. This must occur at either $x=0$, $x=10$ or at a critical point of $s(x)$.

$$s'(x) = 2x + 2(10-x)\cdot(-1)$$
$$= 4x - 20$$

So there is a critical point when $0=4x-20$, that is, at $x=5$.

Now we see that $s(0)=s(10)=100$ and $s(5)=50$

Thus x and y = 0 and 10.